Topic: Set Theory

Introduction:

Sets :

A set is a collection of objects or elements

Examples:

A = {apple, orange, banana, mango}

B = {1, 2, 3, 4, 5}

C = {table, chair, bench, desk}

Note:

1. Preferably denote set with an upper case character

2. Ordering of elements in a set is not important

Notations :

A = {a1, a2, a3, a4, a5} Type equation here.

B = {b1, b2, b3, b4}

A contains 5 elements, a1, a2, a3, a4, a5

$a1 \in A$	$b1 \notin A$
$a2 \in A$	$b1 \in B$
$a3 \in A$	$b2 \in B$
$a4 \in A$	$b3 \in B$
$a5 \in A$	$b4 \in B$

Special Sets

Null Set: A set with no elements Denoted as ∅

Universal Set: A set containing all elements under consideration Denoted as Ω

Standard Sets

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Natural Numbers with 0: $N^0 = \{0, 1, 2, 3, 4, \ldots\}$

Natural Numbers without 0: $N^+ = \{1, 2, 3, 4, \ldots\}$

Integers: $Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Positive Integers: $Z^+ = \{1, 2, 3, 4, \ldots\}$

Negative Integers: $Z^- = \{\ldots, -4, -3, -2, -1\}$

Rational Number: $Q = \{a/b \mid a \in Z, b \in Z \text{ and } b \neq 0\}$

Real Number: $R = \{r \mid -\infty<r<\infty\}$

Complex Numbers: $C = \{z \mid z = a + bi, -\infty<a<\infty, -\infty<b<\infty\}$

Subsets

A set “A” is said to be a subset of another set “B”, if all the elements of set “A” are also elements of B, i.e. $a \in A$

$\Rightarrow a \in B$

Notation: $A \subseteq B$ or “A is a subset of B”

Proper Subset: A set “A” is said to be a proper subset of another set “B”, if all the elements of set “A” are also elements of “B”, and in addition there exists at least one element in “B” that is not in “A”

Notation: $A \subset B$ or “A is a proper subset of B”

Subsets

Examples:

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5\}$ $A \subseteq B$ (also $A \subset B$)

$A = N^0$ $B = Z$ $A \subseteq B$ (also $A \subset B$)

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5\}$ $A \subseteq B$ (also $A \subset B$)

$A = N^0$ $B = Z^+$ $A \not\subset B$ (not a subset)

If $A \subseteq B$ one can also write that B is a superset of A ($B \supseteq A$)

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If $A \subset B$ one can also write that B is a proper superset of A ($B \supset A$)

If $A \not\subset B$ one can also write that B is not a superset of A ($B \not\supset A$)

Set Equality

If two sets contain exactly the same elements then they are said to be equal.

Examples:

A = {cat, dog, mouse} B = {dog, mouse, cat} A = B

A = {cat, dog, mouse} B = {dog, mouse, cat, rat} A ≠ B

If $A \subseteq B$ and $B \subseteq A$, then it implies that A = B

Is this possible, $A \subset B$ and $B \subset A$?

Cardinality

If a set contains "n" distinct countable elements then the cardinality of the set is said to be "n"

Example:

A = {laptop, smartphone, tablet} |A| = 3

B = {{pigeon, sparrow}, {lion, tiger}, rat} |B| = 3

C = {x | x > 1} |C| = Infinite

D = ∅ |D| = 0

Power Set

The power set of a set "A" is another set "P(A)" that contains all the possible subsets of "A"

Example:

A = {a, b, c}

P (A) = {∅, a, b, c, {a, b}, {a, c}, {b, c}, {a, b, c}}

If cardinality of A is n, i.e. |A| = n, then the cardinality of the power set P(A) is 2^n, i.e.

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$|P(A)| = 2^{|A|}$

How do we get this? We will get to know while studying combinatorics.

Ordered n-tuple

The ordered n-tuple (x1, x2, x3, …, xn) is a sequence or ordered collection of “n” objects.

Note:

1. A tuple may contain multiple instances of objects, i.e (5, 2, 2, 3) ≠ (5, 2, 3)

2. A tuple always contains finite elements

3. For sets, {A, B, C} = {C, B, A}, but for tuples, (A, B, C) ≠ (C, B, A)

Cartesian Product

The Cartesian product of two sets “A” and “B” is defined as: A×B = {(a, b) | a ∈ A, b ∈ B}

Example:

A = {a1, a2, a3} and B = {b1, b2}

Then A×B = {(a1, b1), (a1, b2), (a2, b1), (a2, b2), (a3, b1), (a3, b3)}

Note: A×B ≠ B×A

Set Union

“A union B” is the set of all elements that are in A, or B, or both.

Notation: A ∪ B

Set Intersection

“A intersection B” is the set of all elements that are present in both A and B

Notation: A ∩ B

Set Complement

“A complement,” or “not A” is the set of all elements that are not present in A

Notation: A^c or A’

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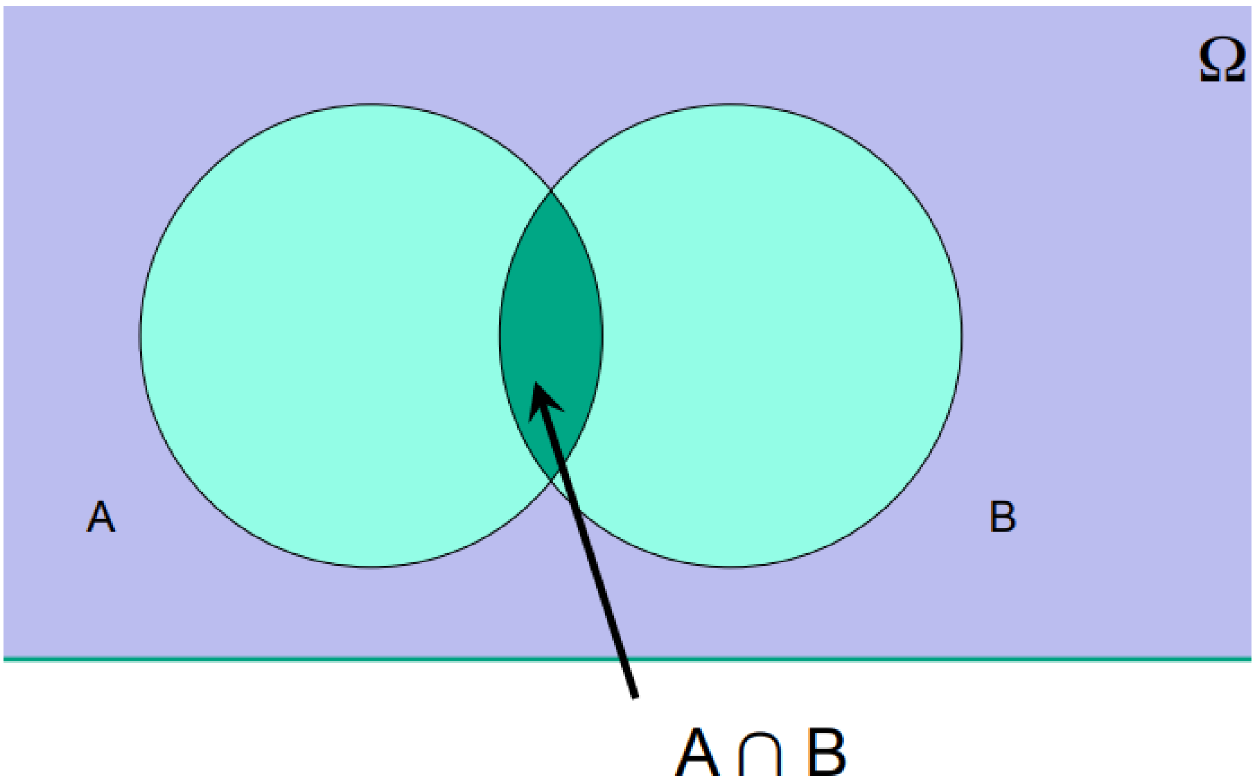
Set Theoretic Difference

The set theoretic difference of sets B and A is the set of elements in B but not in A

Notation: $B - A$ or $B \setminus A = \{x \in B \mid x \notin A\}$

Venn Diagram: Intersection



Venn Diagram: Union

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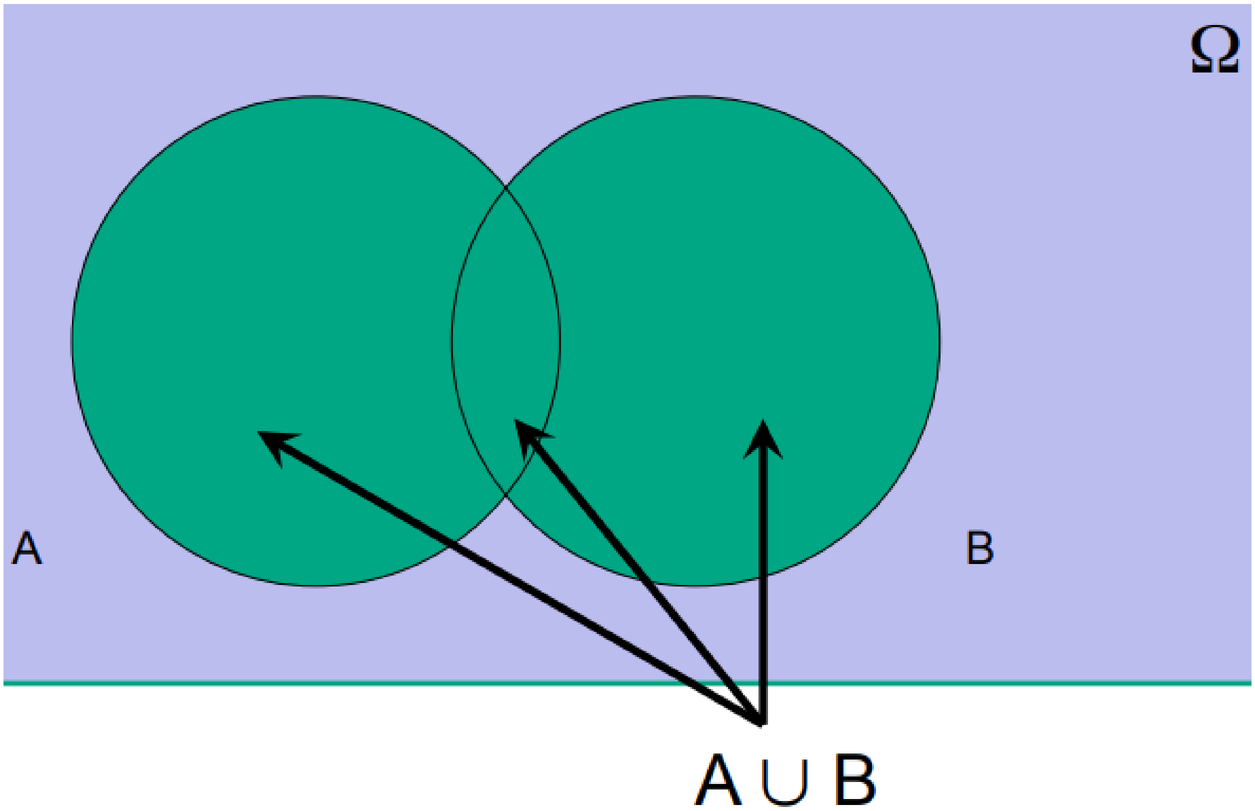

Venn Diagram: Complement

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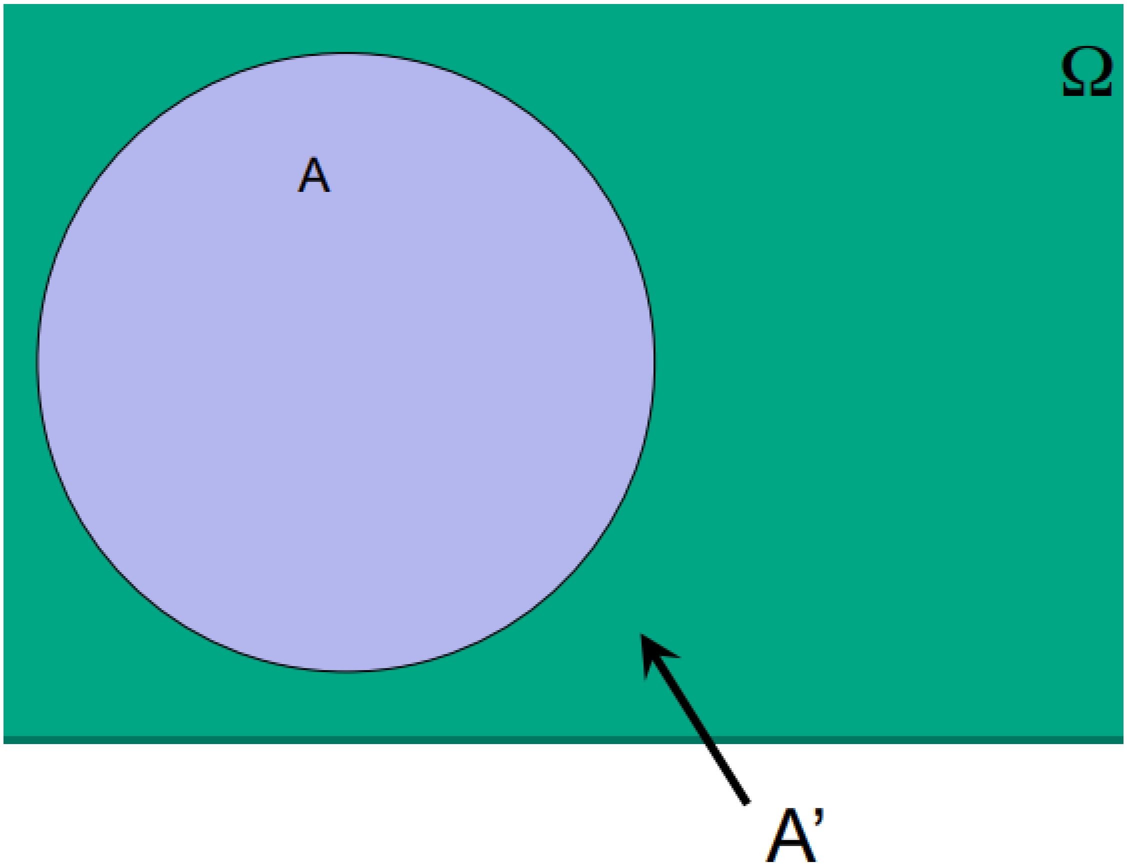

Venn Diagram: Universal Set

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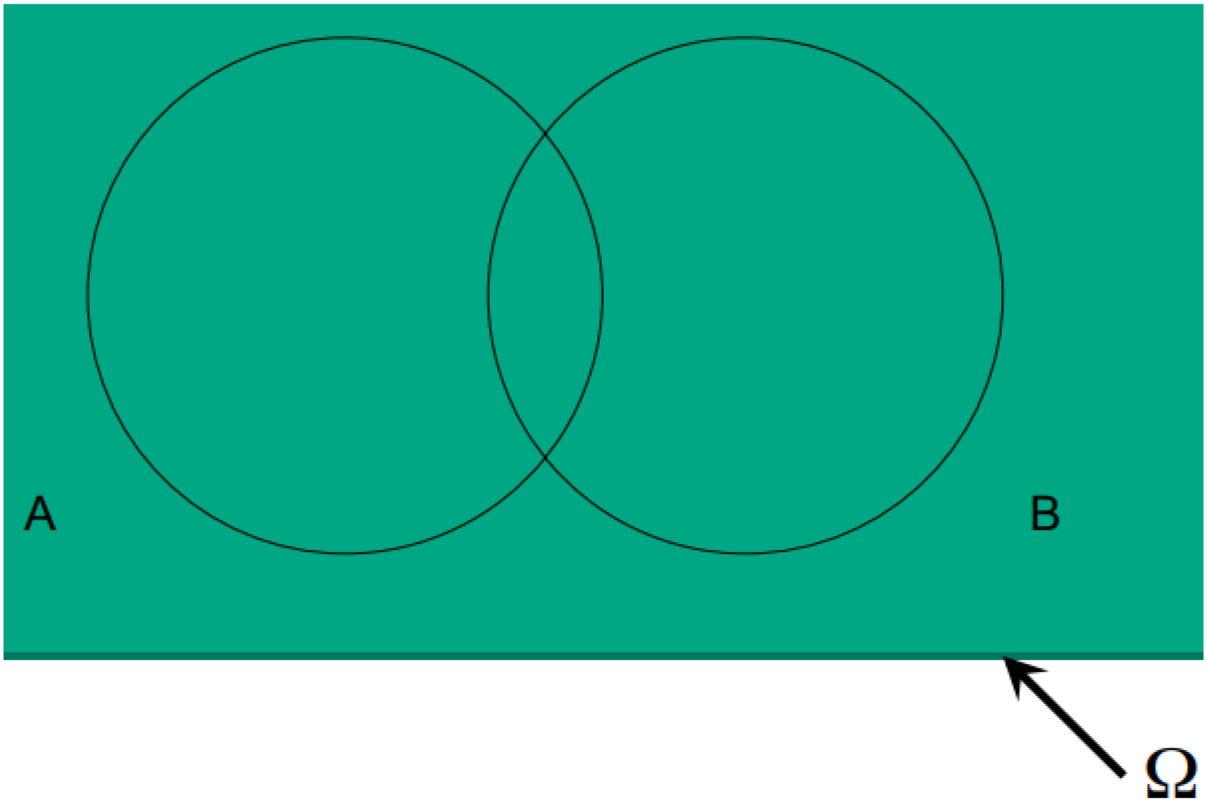

Venn Diagram: Subset

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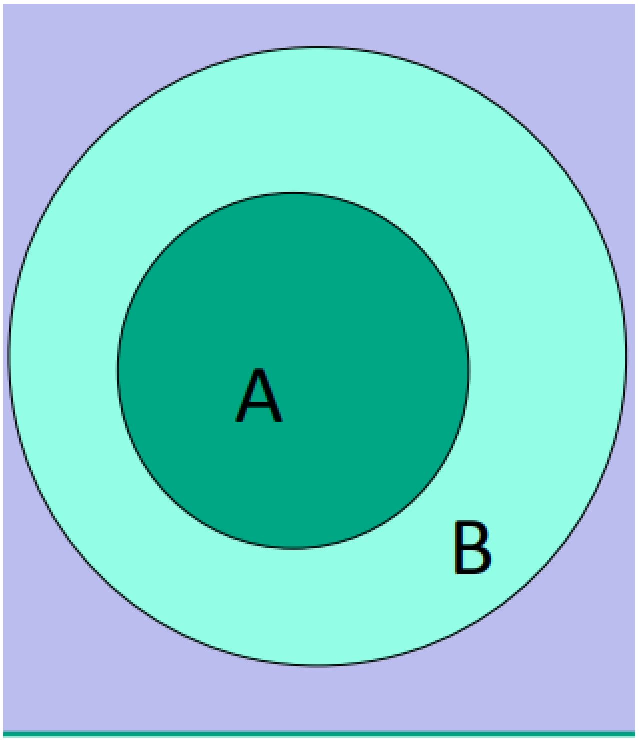
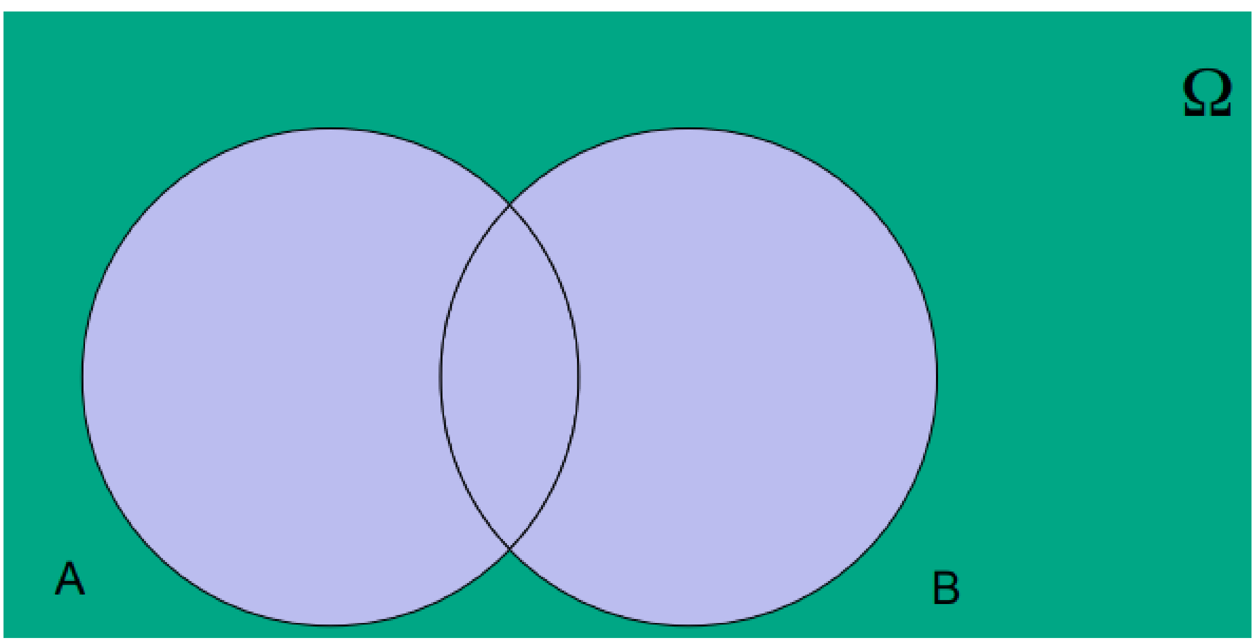


$A \subseteq B$

Venn Diagram: Examples



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$(A \cup B)'$, $A' \cap B'$, $(A \cup B)' = A' \cap B'$, $(A \cap B)'$, $A' \cup B'$, $(A \cap B)' = A' \cup B'$, and $A \cup B'$

Set Theory Rules

• **Commutative Laws:**

$A \cup B = B \cup A$

$A \cap B = B \cap A$

• **Associative Laws:**

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

• **Distributive Laws:**

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

• **DeMorgan's Laws:**

$(A \cap B)' = (A)' \cup (B)'$

$(A \cup B)' = (A)' \cap (B)'$

• $|A \cup B| = |A| + |B| - |A \cap B|$