Topic: Co-ordinate Geometry

Introduction:

A system of geometry where the position of points on the plane is described using an ordered pair of numbers. Recall that a plane is a flat surface that goes on forever in both directions. If we were to place a point on the plane, coordinate geometry gives us a way to describe exactly where it is by using two numbers.

What are coordinates?

Grid with rows and columns labelled. To introduce the idea, consider the grid on the right. The columns of the grid are lettered A,B,C etc. The rows are numbered 1,2,3 etc from the top. We can see that the X is in box D3; that is, column D, row 3.

	A	B	C	D	E	F
1						
2						
3				X		
4						
5						
6						

D and 3 are called the coordinates of the box. It has two parts: the row and the column. There are many boxes in each row and many boxes in each column. But by having both we can find one single box, where the row and column intersect.

The Coordinate Plane

In coordinate geometry, points are placed on the "coordinate plane" as shown below. It has two scales - one running across the plane called the "x axis" and another a right angles to it called the y axis. (These can be thought of as similar to the column and row in the paragraph above.) The point where the axes cross is called the origin and is where both x and y are zero.

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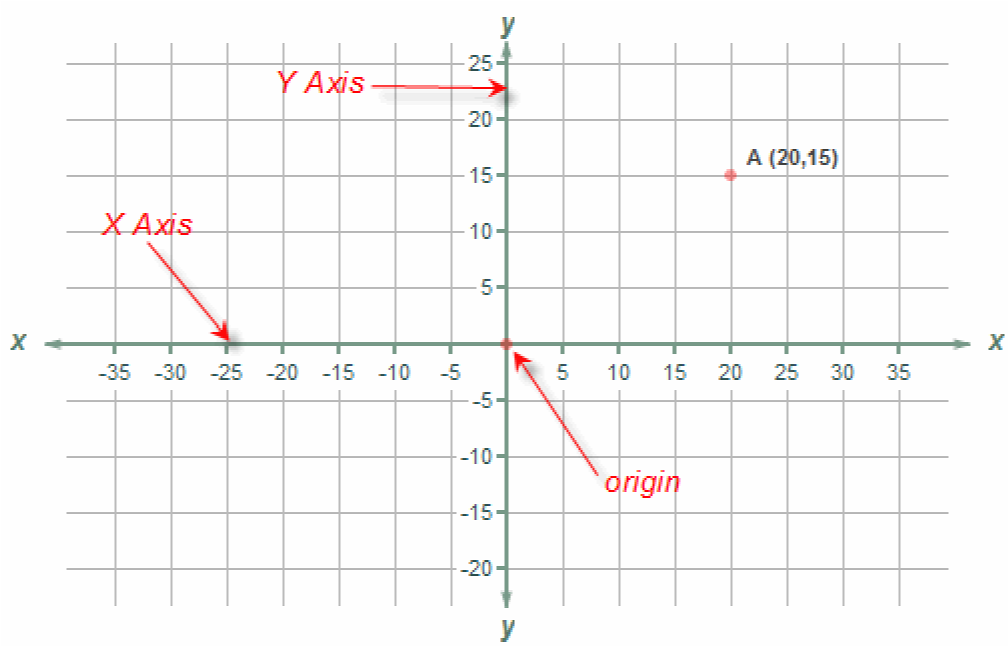

coordinate plane showing x-axis, y-axis and origin

On the x-axis, values to the right are positive and those to the left are negative.

On the y-axis, values above the origin are positive and those below are negative.

A point's location on the plane is given by two numbers,the first tells where it is on the x-axis and the second which tells where it is on the y-axis. Together, they define a single, unique position on the plane. So in the diagram above, the point A has an x value of 20 and a y value of 15. These are the coordinates of the point A, sometimes referred to as its "rectangular coordinates". Note that the order is important; the x coordinate is always the first one of the pair.

History

The method of describing the location of points in this way was proposed by the French mathematician René Descartes (1596 - 1650). (Pronounced "day CART"). He proposed further that curves and lines could be described by equations using this technique, thus being the first to link algebra and geometry. In honor of his work, the coordinates of a point are often referred to as its Cartesian coordinates, and the coordinate plane as the Cartesian Coordinate Plane.

Read More

In two dimensional Coordinate Geometry, location of any point lying in the plane , is

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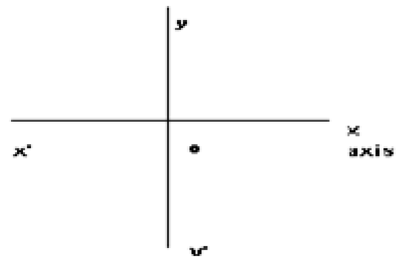
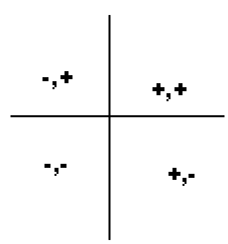
given by specifying the perpendicular distances of the point, from a set of fixed mutually perpendicular lines. The fixed mutually perpendicular lines are known as X-axes and Y-axes respectively. The point of intersection is known as the origin 'O'.



These 2 lines divide the given plane in 4 parts known as quadrants. Distances measured to the right hand side of origin O are treated as positive and distances measured towards the left of origin are treated as negative. In a similar way distances measured along the Y-axis and above the X-axis are treated as positive distance measured below the X-axis are treated as positive distances measured below the X-axis are treated as negative.

The Coordinates of a point are specified as an ordered pair, Comprising of its distance measured along the X and Y axis . Distance measured along one X-axis from the origin is known as abscissa, and usually denoted X, the G distance measured along the Y-axis From the origin is called the ordinate of the point and is denoted by y. Thus coordinate is of a point are specified as (x,y) I.e as (abscissa, ordinate).

The 4 quadrants are known as 1 st ,2 nd 3 rd & 4 th quadrant. The below diagram summarizes the sign of the abscissa and the ordinate of x points lying in 1 st , 2 nd , 3 rd, 4 th quadrant.



Distance Formula

The distance d between 2 points P_1 and P_2 having coordinates

(X_1, Y_1) and (X_2, Y_2) is given by D = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Section Formula and Mid-Point Formula

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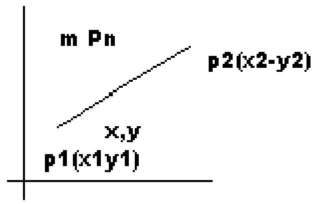


The coordinates P (x,y) of a point which divides the join of points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ in the

ratio m:n is given by X = $\frac{mx_2+nx_1}{n+n}$; Y = $\frac{my_2+ny_1}{m+n}$

Mid-Point Formula

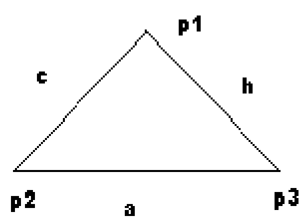
The Coordinates of the mid-point of the line joining $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by

X = $\frac{x_1+x_2}{2}$; Y = $\frac{y_1+y_2}{2}$

Coordinates of the centroid of a triangle

Let $P_1(x_1,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ be the coordinates of the vertices of triangle $P_1P_2P_3$, then coordinates of it's centroid (x,y) is given by

X = $\frac{y_1+y_2+y_3}{3}$; Y = $\frac{y_1+y_2+y_3}{3}$



The coordinates of the incentre I (x,y) of the triangle $P_1P_2P_3$ with vertices $P_1(x_1,y_1)$; $P_2(x_2,y_2)$

$P_3(x_3,y_3)$ and side length a,b,c is given by X = $\frac{ax_1+bx_2+cx_3}{a+b+c}$ $\frac{ay_1+by_2+cy_3}{a+b+c}$

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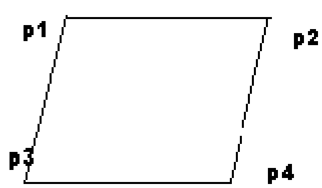
Area of a triangle

The area of triangle having vertices $P_1(x_1, y_1)$; $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ is given by = $\frac{1}{2}$ [$(x_1y_2 - x_2y_1)$ + $(x_2y_3 - x_3y_2)$ + $(x_3y_1 - x_1y_3)$]

Note – If area of is Zero $\Rightarrow$ points P_1, P_2, P_3 are collinear.

Condition for 4 points , no three of which are collinear to be a parallelogram . Let $p_1(x_1, y_1)$, $p_2(x_2, y_2)$, $p_3(x_3, y_3)$, $p_4(x_4, y_4)$ be 4 points lying in a plane then p_1 , p_2 , p_3 , p_4 are the verticals of a parallelogram if $x_1 + x_3 = x_2 + x_4$ and $y_1 + y_3 = y_2 + y_4$

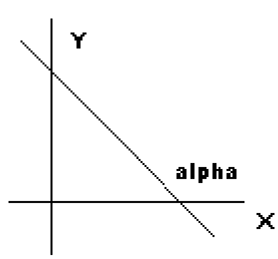


p_1, p_2, p_3, p_4 are the vertices of a square if in addition to above or $x_4 - x_1 = y_3 - y_2, y_4 - y_1 = x_3 - x_2$

Equation of a Line

Slope of a line

The slope of a line is defined as tangent of the angle α which the line makes in the positive direction of the x-axis in the anti – clockwise direction.



The slope is generally represented by 'm' thus in the notion used above $m = \tan\alpha$

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The equation of a line is given by y = mx+c where 'c' is the intercept mode on the y -axis by the line.

Equation of x-axis is y = 0

Equation of y-axis is x = 0

2 lines are said to be parallel if they have the same slope that is if where $m_1=m_2$ where m1 and m2 are the slopes of the 2lines.

2 lines said to be perpendicular if they product of there slope is -1. that is if $m_1 . m_2 = -1$ then lines are perpendicular to each other. Different forms of the equation of a line.

Line passing through point $p(x_1, y_1)$ and having slope 'm' .

$(y-y_1)=m(x-x_1)$

Line passing through points $p_1(x_1, y_1)$ and $p_2(x_2, y_2)$.

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

If a line makes intercept a,b on x and y-axis respectively then equation of line is given by $\frac{x}{a}+\frac{y}{b}=1$

Angle between 2lines having slopes m_1 and m_2 is given by $\theta=\tan^{-1}\left(\left|\frac{m_1-m_2}{1+m_1m_2}\right|\right)$ where θ is the actual angle between the 2 lines.

Perpendicular distance between parallel lines , Let the equation of 2 lines be $y=mx+c_1$ and line is given by $\left|\frac{c_1-c_2}{\sqrt{1+m^2}}\right|$

Perpendicular distance of a point from a line . Let $p(x_1, y_1)$ be any point and let y = mx +c be any line, then the perpendicular distance of point p from line y = mx + c is given by $\left|\frac{y_1-mx_1-c}{\sqrt{1+m_2}}\right|$

Equation of a Circle

The equation of a circle having center at point (h,x) and radius 'r' given by $(x-h)^2+(y-x)^2=r^2$

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Solved Examples

Question

Find the equation of the line with slope 2 and intercept on the y-axis as -7 ?

Solution

we know that equation of the line having slope 'm' and intercept on y-axis is 'c' the equation is y = mx +c , hence equation of line is y = 2x – 7

Question

The equation of a line which makes an intercept of 3 on x-axis and -3 on y-axis is

x – 2y = 5

x – 2y = 3

4x + 13y = 7

none of these

Solution

The line with the x-axis at point (3,0) and y-axis at the point (0,-3) hence equation of line is using

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{-3 - 0}{0 - 3}(x - 3)$$

y = x-3 or x – y = 3

hence , answer is option (2)

Question

Find the equation of the line perpendicular to the line x + y = 2 and passing through (1,2)

y = x + 3

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y = x – 1

y = x + 1

y – x = 2

Solution

Let slope of required line be 'm' then slope of line x + y = 2 is '-1' for lines to be perpendicular the product of their slope should be '-1' hence we get $m \times (-1) = -1$ m = 1 using one point slope from the equation of we get required equation as (y – 2) = 1. (x-1) y = x + 1

Hence (3) is the answer.

Question

The equation of the line through the intersection of the lines 2x + y = 3 and 3y – x + 2 = 0 and having slope -1/2 is

14y +5x +10 = 0

14y+7x-9 = 0

14y+7x-6 = 0

14y+7x+11 = 0

Solution

Equation of any line through the point of intersection of the given lines is of the form (2x+y-3) + (3y-x+2) = 0 where k is the constant to be determined , remaining we get (3k+1)y + (2-k)x+2k-3 = 0,but slope of this line $\frac{k-2}{3k+1} = \frac{-1}{2}$ (given) solving we get k = 3/5

Hence putting the value of k = 3/5 we get equation required line as 14y+7x-9 = 0

Question

The coordinate of the verities of a triangle are

1. (3,5)

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2. (2,6)

3. (4,4)

4. (2,4)

Solution

The Coordinate of the centroid of the triangle with vertical $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by X = $\frac{x_1+x_2+x_3}{3}$: Y = $\frac{y_1+y_2+y_3}{3}$

Hence X = $\frac{2+3+1}{3}=2$ Y = $\frac{3+5+4}{3}=4$

Therefore option (4) is the correct answer.

More About :

Few questions on coordinate geometry. These are slightly unconventional questions. CAT has stayed away from raw, brute-force cogeo questions for a while now. The questions that we see in CAT have not been algebraic. They have either been of the visual type or of the type where one has to do trial and error. So, the cogeo basics are important, but no point learning all kinds of fancy formula stuff.

1. Set S contains points whose abscissa and ordinate are both natural numbers. Point P, an element in set S has the property that the sum of the distances from point P to the point (3,0) and the point (0,5) is the lowest among all elements in set S. What is the sum of abscissa and ordinate of point P?

2. Region R is defined as the region in the first quadrant satisfying the condition $3x + 4y < 12$. Given that a point P with coordinates (r, s) lies within the region R, what is the probability that $r > 2$?

3. Region Q is defined by the equation $2x + y < 40$. How many points (r, s) exist such that r is a natural number and s is a multiple of r?

Have given the solutions to questions on Cogeo

1. Set S contains points whose abscissa and ordinate are both natural numbers. Point P, an element in set S has the property that the sum of the distances from point P to the point (3,0) and the point (0,5) is the lowest among all elements in set S. What is the sum

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of abscissa and ordinate of point P?

Any point on the line x/3 + y/5 =1 will have the shortest overall distance. However, we need to have integral coordinates. So, we need to find points with integral coordinates as close as possible to the line 5x + 3y =15.

Substitute x =1, we get y = 2 or 3

Substitute x = 2, we get y = 1 or 2

Sum of distances for (1, 2) = sqrt(8) + sqrt (10)

Sum of distances for (1, 3) = sqrt(13) + sqrt (5)

Sum of distances for (2, 1) = sqrt(2) + sqrt (20)

Sum of distances for (2, 2) = sqrt(5) + sqrt (13)

sqrt(5) + sqrt (13) is the shortest distance. Sum of abscissa + ordinate = 4

2. Region R is defined as the region in the first quadrant satisfying the condition 3x + 4y < 12. Given that a point P with coordinates (r, s) lies within the region R, what is the probability that r > 2?

Line 3x + 4y =12 cuts the x-axis at (4, 0) and y axis at (0, 3)

The region in the first quadrant satisfying the condition 3x + 4y < 12 forms aright triangle with sides 3, 4 and 5. Area of this triangle = 6 sq units.

The lines x = 2 and 3x + 4y = 12 intersect at (2, 1.5). So, the region r > 2, 3x + 4y < 12 also forms a right triangle. This right triangle has base sides 2, 1.5. Area of this triangle = 1.5

Probability of the point lying in said region = 1.5/6 = 1/4

3. Region Q is defined by the equation 2x + y < 40. How many points (r, s) exist such that r is a natural number and s is a multiple of r?

When r = 1, s can take 37 values [37/1]

When r = 2, s can take 17 values [35/2]

When r = 3, s can take 11 values [33/3]

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When r = 4, s can take 7 values [31/4]

When r = 5, s can take 5 values [29/5]

When r = 6, s can take 4 values [27/6]

When r = 7, s can take 3 values [25/7]

When r = 8, s can take 2 values [23/8]

When r = 9, s can take 2 values [21/9]

When r = 10, s can take 1 values [19/10]

When r = 11, 12, 13 s can take 1 values one value each

Totally, there are 92 values possible.