Topic: Polynomials

Introduction:

Polynomials = Poly (means many) + nomials (means terms). Thus, a polynomial contains many terms.Thus, a type of algebraic expression with many terms having variables and coefficients is called polynomial.

A polynomial is an expression consisting of variables (or indeterminates) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables. An example of a polynomial of a single indeterminate x is $x^2 - 4x + 7$. An example in three variables is $x^3 + 2xyz^2 - yz + 1$.

Polynomials appear in a wide variety of areas of mathematics and science. For example, they are used to form polynomial equations, which encode a wide range of problems, from elementary word problems to complicated problems in the sciences; they are used to define polynomial functions, which appear in settings ranging from basic chemistry and physics to economics and social science; they are used in calculus and numerical analysis to approximate other functions. In advanced mathematics, polynomials are used to construct polynomial rings and algebraic varieties, central concepts in algebra and algebraic geometry.

A real polynomial is a polynomial with real coefficients. The argument of the polynomial is not necessarily so restricted, for instance the s-plane variable in Laplace transforms. A real polynomial function is a function from the reals to the reals that is defined by a real polynomial.

A polynomial in one indeterminate is called a **univariate polynomial**, a polynomial in more than one indeterminate is called a **multivariate polynomial**. A polynomial with two indeterminates is called a **bivariate polynomial**. These notions refer more to the kind of polynomials one is generally working with than to individual polynomials; for instance when working with univariate polynomials one does not exclude constant polynomials (which may result, for instance, from the subtraction of non-constant polynomials), although strictly speaking constant polynomials do not contain any indeterminates at all. It is possible to further classify multivariate polynomials as bivariate, trivariate, and so on, according to the maximum number of indeterminates allowed. Again, so that the set of objects under consideration be closed under subtraction, a study of trivariate polynomials usually allows bivariate polynomials, and so on. It is common, also, to say simply "polynomials in x, y, and z", listing the indeterminates allowed.

Example :

Topic: Polynomials

$3x$

$5y^2 + 2x + 5$

$2x^2 + 2$

Let us consider the second example – $5x^2 + 2y + 5$ .In this there are two variables, i.e. x and y. Such polynomials with two variables are called **Polynomials of two variables**.

Power of x is 2. This means exponent of x is 2. Power of y is 1. This means exponent of y is 1. The term '5' is constant.

Let us consider third example, $2x^2 + 2$ in this 'x' is called variable.Power of 'x', i.e. 2 is called exponent. Multiple of 'x', i.e. 2 is called coefficient. The term '2' is called constant. And all items are called terms.

There are three terms in this polynomial.

Types of Polynomial:

Monomial – Algebraic expression with only one term is called monomial.

Example: $2x, 2, 5x, 3y, etc.$

Binomial – Algebraic expression with two terms is called binomial.

Example: $2x + 2, 3y^2 + 5, 3m + 3, etc.$

Trinomial – Algebraic expression with three terms is called trinomial.

Example: $3x + 2y + 2, 5y^2 + 2y + 2, etc.$

But algebraic expressions having more than two terms are collectively known as polynomials.

Variables and Polynomial:

Polynomial of zero variable :

If a polynomial has no variable, it is called polynomial of zero variable. For example – 5. This polynomial has only one term, which is constant.

Topic: Polynomials

Polynomial of one variable – Polynomial with only one variable is called Polynomial of one variable.

Example : $5x + 2, 2x^2 + x + 3, etc$

In the given example polynomials have only one variable i.e. x, and hence it is a polynomial of one variable.

Polynomial of two variables – Polynomial with two variables is known as Polynomial of two variables.

Example – $5x + y, 2x + 3y + 2, etc.$

In the given examples polynomials have two variables, i.e. x and y, and hence are called polynomial of two variables.

Polynomial of three variables – Polynomial with three variables is known as Polynomial of three variables.

Example –

$5x + 2y + z$

$2x^2 + 3y + m + 2$

$5y + 3m + z + 5$

In the above examples polynomials have three variables, and thus are called Polynomials of three variables.

In similar way a polynomial can have of four, five, six, …. etc. variables and thus are named as per the number of variables.

Degree of Polynomials:

Highest exponent of a polynomial decides its degree.

Polynomial of 1 degree:

Example: 2x + 1

In this since, variable x has power 1, i.e. x has coefficient equal to 1 and hence is called

Topic: Polynomials

polynomial of one degree.

Polynomial of 2 degree:

Example: class nine math number system polynomial 10.

In this expression, exponent of x in the first term is 2, and exponent of x in second term is 1, and thus, this is a polynomial of two(2) degree.

To decide the degree of a polynomial having same variable, the highest exponent of variable is taken into consideration.

Similarly, if variable of a polynomial has exponent equal to 3 or 4, that is called polynomial of 3 degree or polynomial of 4 degree respectively.

Generalizations:

There are several generalizations of the concept of polynomials.

Trigonometric polynomials:

A trigonometric polynomial is a finite linear combination of functions sin(nx) and cos(nx) with n taking on the values of one or more natural numbers. The coefficients may be taken as real numbers, for real-valued functions. If sin(nx) and cos(nx) are expanded in terms of sin(x) and cos(x), a trigonometric polynomial becomes a polynomial in the two variables sin(x) and cos(x) . Conversely, every polynomial in sin(x) and cos(x) may be converted, with Product-to-sum identities, into a linear combination of functions sin(nx) and cos(nx). This equivalence explains why linear combinations are called polynomials.

For complex coefficients, there is no difference between such a function and a finite Fourier series.

Trigonometric polynomials are widely used, for example in trigonometric interpolation applied to the interpolation of periodic functions. They are used also in the discrete Fourier transform.

Matrix polynomials:

A matrix polynomial is a polynomial with matrices as variables. Given an ordinary, scalar -valued polynomial

A matrix polynomial equation is an equality between two matrix polynomials, which holds for the specific matrices in question. A matrix polynomial identity is a matrix

Topic: Polynomials

polynomial equation which holds for all matrices A in a specified matrix ring $M_n(R)$.

Laurent polynomials:

Laurent polynomials are like polynomials, but allow negative powers of the variable(s) to occur.

Rational functions :

A rational fraction is the quotient (algebraic fraction) of two polynomials. Any algebraic expression that can be rewritten as a rational fraction is a rational function. While polynomial functions are defined for all values of the variables, a rational function is defined only for the values of the variables for which the denominator is not zero. The rational fractions include the Laurent polynomials, but do not limit denominators to powers of an indeterminate.

Polynomial functions:

A polynomial function is a function that can be defined by evaluating a polynomial. A function f of one argument is thus a polynomial function if it satisfies.

Polynomial functions are a class of functions having many important properties. They are all continuous, smooth, entire, computable, etc.

Graphs:

A polynomial function in one real variable can be represented by a graph.

- The graph of the zero polynomial

  $f(x) = 0$,is the x-axis.

- The graph of a degree 0 polynomial

  $f(x) = a_0$, where $a_0 \neq 0$, is a horizontal line with y-intercept a_0

- The graph of a degree 1 polynomial (or linear function)

  $f(x) = a_0 + a_1x$, where $a_1 \neq 0$, is an oblique line with y-intercept a_0 and slope a_1.

- The graph of a degree 2 polynomial

  $f(x) = a_0 + a_1x + a_2x_2$, where $a_2 \neq 0$, is a parabola.

Topic: Polynomials

- The graph of a degree 3 polynomial $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, where $a_3 \neq 0$, is a cubic curve.
- The graph of any polynomial with degree 2 or greater

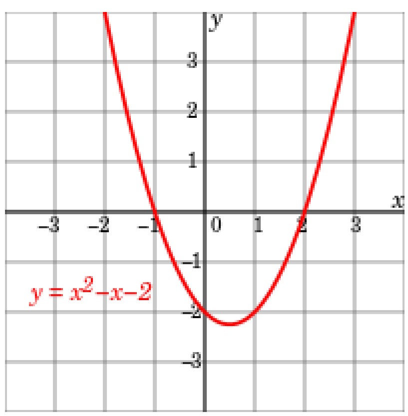
  $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where $a_n \neq 0$ and $n \geq 2$,is a continuous non-linear curve.

A non-constant polynomial function tends to infinity when the variable increases indefinitely (in absolute value). If the degree is higher than one, the graph does not have any asymptote. It has two parabolic branches with vertical direction (one branch for positive x and one for negative x).

Polynomial graphs are analyzed in calculus using intercepts, slopes, concavity, and end behavior.

- Polynomial of degree 2:

  $f(x) = x^2 - x - 2 = (x + 1)(x - 2)$



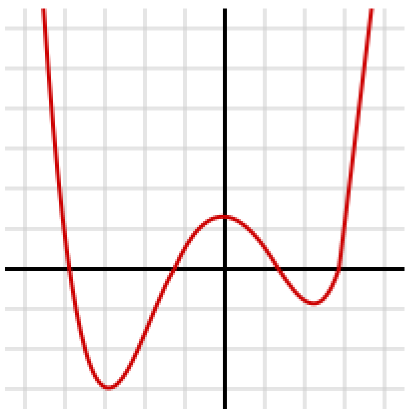
- Polynomial of degree 3:

  $f(x) = x^3/4 + 3x^2/4 - 3x/2 - 2 = 1/4\ (x + 4)(x + 1)(x - 2)$

Topic: Polynomials

- Polynomial of degree 4:

  $f(x) = 1/14\ (x + 4)(x + 1)(x - 1)(x - 3) + 0.5$



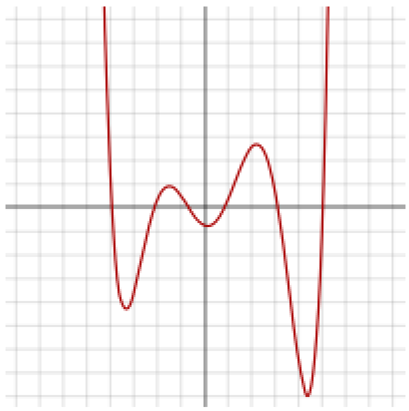
- Polynomial of degree 5:

  $f(x) = 1/20\ (x + 4)(x + 2)(x + 1)(x - 1)(x - 3) + 2$

Topic: Polynomials

- Polynomial of degree 6:

$f(x) = 1/100\,(x^6 - 2x^5 - 26x^4 + 28x^3 + 145x^2 - 26x - 80)$



- Polynomial of degree 7:

$f(x) = (x - 3)(x - 2)(x - 1)(x)(x + 1)(x + 2)(x + 3)$

Important points about Polynomials:

- A polynomial can have many terms but not infinite terms.
- Exponent of a variable of a polynomial cannot be negative. This means, a variable with power - 2, -3, -4, etc. is not allowed. If power of a variable in an algebraic expression is negative, then that cannot be considered a polynomial.
- The exponent of a variable of a polynomial must be a whole number.
- Exponent of a variable of a polynomial cannot be fraction. This means, a variable with power 1/2, 3/2, etc. is not allowed. If power of a variable in an algebraic expression is in fraction, then that cannot be considered a polynomial.
- Polynomial with only constant term is called constant polynomial.

Topic: Polynomials

- The degree of a non-zero constant polynomial is zero.
- Degree of a zero polynomial is not defined.
- A sum of polynomials is a polynomial.
- A product of polynomials is a polynomial.
- A composition of two polynomials is a polynomial, which is obtained by substituting a variable of the first polynomial by the second polynomial.