Topic: Mensuration

Introduction:

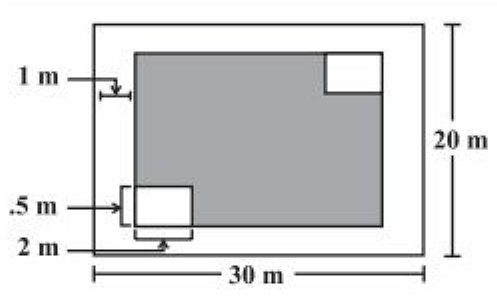
We have learnt that for a closed plane figure, the perimeter is the distance around its boundary and its area is the region covered by it. We found the area and perimeter of various plane figures such as triangles, rectangles, circles etc. We have also learnt to find the area of pathways or borders in rectangular shapes.

In this chapter, we will try to solve problems related to perimeter and area of other plane closed figures like quadrilaterals.

We will also learn about surface area and volume of solids such as cube, cuboid and cylinder.

Let us Recall

Let us take an example to review our previous knowledge. This is a figure of a rectangular park (Fig) whose length is 30 m and width is 20 m.



(i) What is the total length of the fence surrounding it? To find the length of the fence we need to find the perimeter of this park, which is 100 m. (Check it)

(ii) How much land is occupied by the park? To find the land occupied by this park we need to find the area of this park which is 600 square meters (m^2) (How?).

(iii) There is a path of one metre width running inside along the perimeter of the park that has to be cemented. If 1 bag of cement is required to cement $4m^2$ area, how many bags of cement would be required to construct the cemented path?

We can say that the number of cement bags used = $\frac{\text{area of the path}}{\text{area cemented by 1 bag}}$.

Area of cemented path = Area of park – Area of park not cemented. Path is 1 m wide, so the rectangular area not cemented is (30 – 2) × (20 – 2) m^2 . That is 28 × 18 m^2 .

Hence number of cement bags used = -----------------

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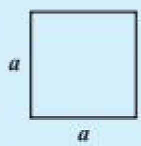
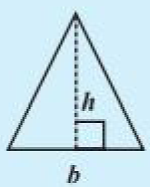
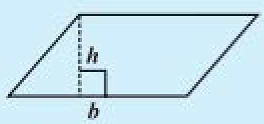
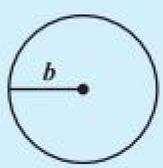
(iv) There are two rectangular flower beds of size 1.5 m × 2 m each in the park as shown in the diagram (Fig) and the rest has grass on it. Find the area covered by grass.

Area of rectangular beds = ----------------- Area of park left after cementing the path = ----------------- Area covered by the grass = ----------------- We can find areas of geometrical shapes other than rectangles also if certain measurements are given to us . Try to recall and match the following:

Diagram	Shape	Area
	rectangle	$a \times a$
	square	$b \times h$
	triangle	πb^2
	parallelogram	$\frac{1}{2} b \times h$
	circle	$a \times b$
Can you write an expression for the perimeter of each of the above shapes?		

Area of Trapezium:

Nazma owns a plot near a main road (Fig 11.2). Unlike some other rectangular plots in her neighbourhood, the plot has only one pair of parallel opposite sides. So, it is nearly a trapezium in shape. Can you find out its area?

Let us name the vertices of this plot as shown in Fig 11.3.

By drawing EC || AB, we can divide it into two parts, one of rectangular shape and the other of

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triangular shape, (which is right angled at C), as shown in Fig 11.3.

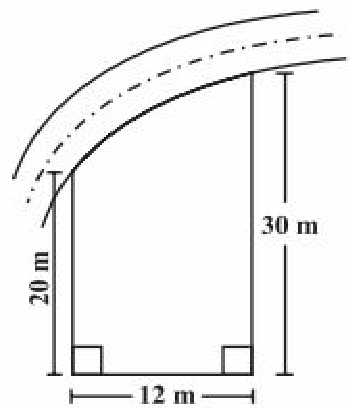

Fig 11.2

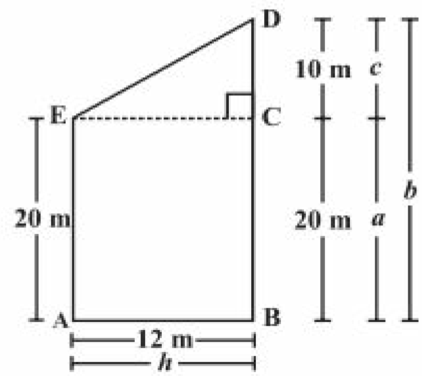

Fig 11.3

($b = c + a = 30$ m)

Area of Δ ECD $= \frac{1}{2} h \times c = \frac{1}{2} \times 12 \times 10 = 60 \text{ m}^2$.

Area of rectangle ABCE $= h \times a = 12 \times 20 = 240 \text{ m}^2$.

Area of trapezium ABDE = area of Δ ECD + Area of rectangle ABCE $= 60 + 240 = 300 \text{ m}^2$.

We can write the area by combining the two areas and write the area of trapezium as

$$\text{area of ABDE} = \frac{1}{2} h \times c + h \times a = h\left(\frac{c}{2} + a\right)$$

$$= h\left(\frac{c + 2a}{2}\right) = h\left(\frac{c + a + a}{2}\right)$$

$$= h\frac{(b+a)}{2} = \text{height } \frac{(\text{sum of parallel sides})}{2}$$

By substituting the values of h, b and a in this expression, we find $h\frac{(b+a)}{2} = 300 \text{ m}^2$.

Area of a General Quadrilateral:

A general quadrilateral can be split into two triangles by drawing one of its diagonals. This "triangulation" helps us to find a formula for any general quadrilateral. Study the Fig 11.10. Area of quadrilateral ABCD

= (area of Δ ABC) + (area of Δ ADC)

$= (\frac{1}{2} \text{ AC} \times h_1) + (\frac{1}{2} \text{ AC} \times h_2)$

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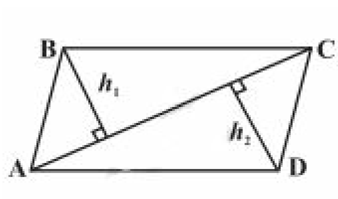
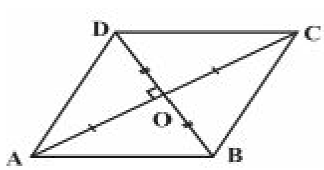
$= \frac{1}{2} AC \times (h_1 + h_2)$

$= \frac{1}{2} d (h1 + h2)$ where d denotes the length of diagonal AC.



Area of special quadrilaterals:

We can use the same method of splitting into triangles (which we called "triangulation") to find a formula for the area of a rhombus. In Fig 11.13 ABCD is a rhombus. Therefore, its diagonals are perpendicular bisectors of each other.



Area of rhombus ABCD = (area of Δ ACD) + (area of Δ ABC)

$= (\frac{1}{2} \times AC \times OD) + (\frac{1}{2} \times AC \times OB)$

$= \frac{1}{2} AC \times (OD + OB)$

$= \frac{1}{2} AC \times BD$

$= \frac{1}{2} d_1 \times d_2$; where $AC = d_1$ and $BD = d_2$

In other words, area of a rhombus is half the product of its diagonals.

Area of a Polygon:

We split a quadrilateral into triangles and find its area. Similar methods can be used to find the area of a polygon. Observe the following for a pentagon:

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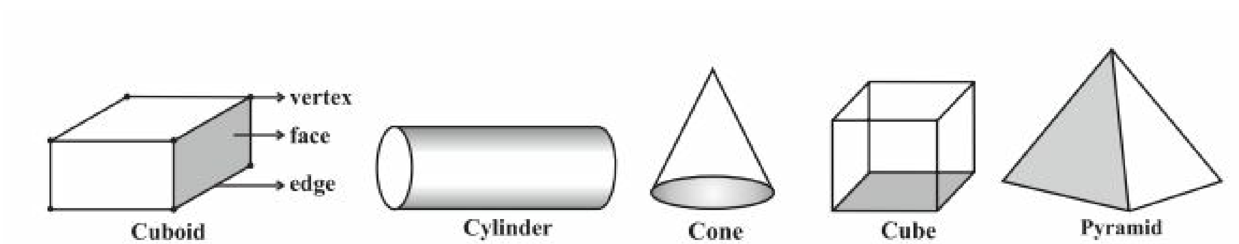
By constructing two diagonals AC and AD the pentagon ABCDE is divided into three parts. So, area ABCDE = area of Δ ABC + area of Δ ACD + area of Δ AED.

By constructing one diagonal AD and two perpendiculars BF and CG on it, pentagon ABCDE is divided into four parts. So, area of ABCDE = area of right angled Δ AFB + area of trapezium BFGC + area of right angled Δ CGD + area of Δ AED. (Identify the parallel sides of trapezium BFGC.)

Solid Shapes:

In your earlier classes you have studied that two dimensional figures can be identified as the faces of three dimensional shapes. Observe the solids which we have discussed so far (Fig).



Observe that some shapes have two or more than two identical (congruent) faces. Name them. Which solid has all congruent faces?

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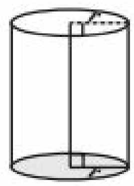


Fig 11.26
(This is a right circular cylinder)

Did you notice the following:

The cylinder has congruent circular faces that are parallel to each other (Fig 11.26). Observe that the line segment joining the center of circular faces is perpendicular to the base. Such cylinders are known as **right circular cylinders**. We are only going to study this type of cylinders, though there are other types of cylinders as well (Fig 11.27).

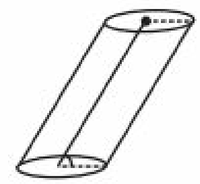
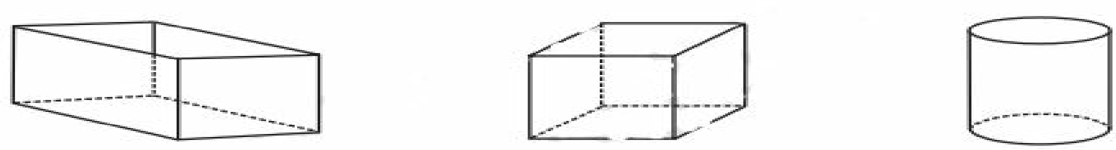


Fig 11.27
(This is not a right circular cylinder)

Surface Area of Cube, Cuboid and Cylinder:

Imran, Monica and Jaspal are painting a cuboidal, cubical and a cylindrical box respectively of same height (Fig 11.28).



They try to determine who has painted more area. Hari suggested that finding the surface area of each box would help them find it out. To find the total surface area, find the area of each face and then add. The surface area of a solid is the sum of the areas of its faces. To clarify further, we take each shape one by one.

Cuboid:

Suppose you cut open a cuboidal box and lay it flat (Fig 11.29). We can see a net as shown below (Fig).

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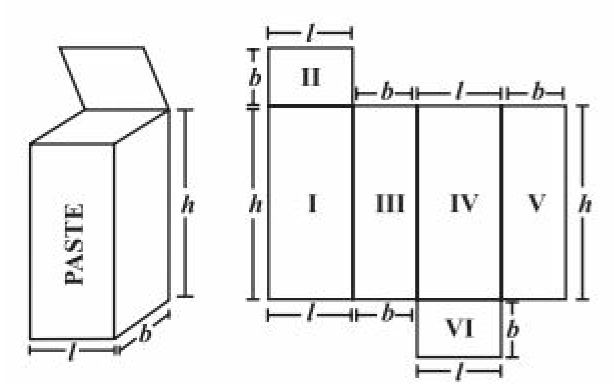


Write the dimension of each side. You know that a cuboid has three pairs of identical faces. What expression can you use to find the area of each face? Find the total area of all the faces of the box. We see that the total surface area of a cuboid is area I + area II + area III + area IV +area V + area VI

$= h \times l + b \times l + b \times h + l \times h + b \times h + l \times b$

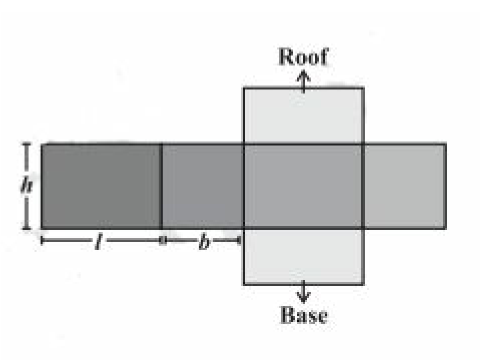
So total surface area = $2(h \times l + b \times h + b \times l) = 2(lb + bh + hl)$

where h, l and b are the height, length and width of the cuboid respectively. Suppose the height, length and width of the box shown above are 20 cm, 15 cm and 10 cm respectively.

Then the total surface area = $2(20 \times 15 + 20 \times 10 + 10 \times 15) = 2(300 + 200 + 150) = 1300\ m^2$

• The side walls (the faces excluding the top and bottom) make the lateral surface area of the cuboid. For example, the total area of all the four walls of the cuboidal room in which you are sitting is the lateral surface area of this room (Fig). Hence, the lateral surface area of a cuboid is given by

$2(h \times l + b \times h)$ or $2h(l + b)$.



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Volume of Cube, Cuboid and Cylinder:

Amount of space occupied by a three dimensional object is called its volume. Try to compare the volume of objects surrounding you. For example, volume of a room is greater than the volume of an almirah kept inside it. Similarly, volume of your pencil box is greater than the volume of the pen and the eraser kept inside it. Can you measure volume of either of these objects? Remember, we use square units to find the area of a region. Here we will use cubic units to find the volume of a solid, as cube is the most convenient solid shape (just as square is the most convenient shape to measure area of a region). For finding the area we divide the region into square units, similarly, to find the volume of a solid we need to divide it into cubical units. Observe that the volume of each of the adjoining solids is 8 cubic units (Fig 11.42). We can say that the volume of a solid is measured by counting the number of unit cubes it contains. Cubic units which we generally use to measure volume are

1 cubic cm = 1 cm × 1 cm × 1 cm = 1 cm^3

= 10 mm × 10 mm × 10 mm = ... mm^3

1 cubic m = 1 m × 1 m × 1 m = 1 m^3

= .... cm^3

1 cubic mm = 1 mm × 1 mm × 1 mm = 1 mm^3

= 0.1 cm × 0.1 cm × 0.1 cm = ... cm^3

We now find some expressions to find volume of a cuboid, cube and cylinder. Let us take each solid one by one. 11.8.1

Cuboid:

Take 36 cubes of equal size (i.e., length of each cube is same). Arrange them to form a cuboid. You can arrange them in many ways. Observe the following table and fill in the blan

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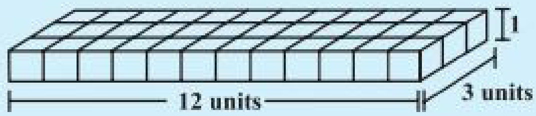
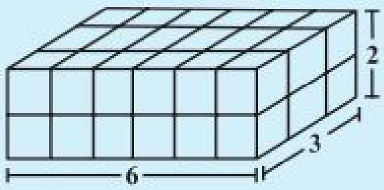
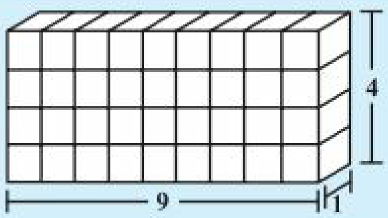
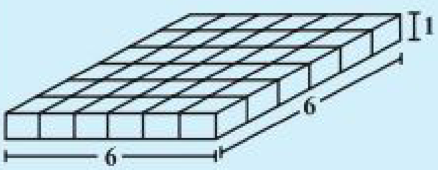
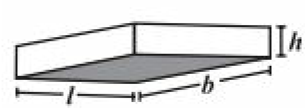
	cuboid	length	breadth	height	$l \times b \times h = V$
(i)		12	3	1	$12 \times 3 \times 1 = 36$
(ii)		...	...	...	...
(iii)		...	...	...	...
(iv)		...	...	...	...

What do you observe? Since we have used 36 cubes to form these cuboids, volume of each cuboid is 36 cubic units. Also volume of each cuboid is equal to the product of length, breadth and height of the cuboid. From the above example we can say



volume of cuboid = l × b × h.

Since l × b,is the area of its base we can also say that,

Volume of cuboid = area of the base × height

Cube

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The cube is a special case of a cuboid, where l = b = h. Hence, volume of cube = l × l × l = l^3

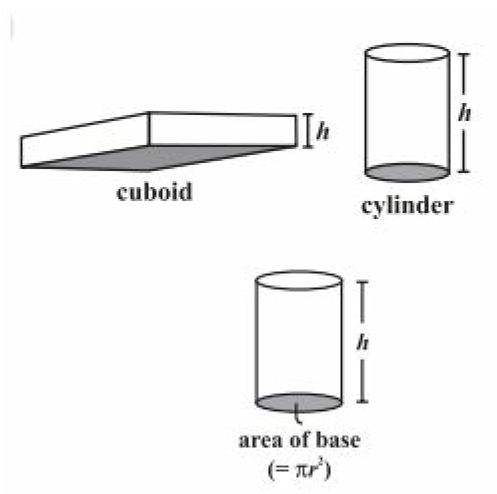
Cylinder:

We know that volume of a cuboid can be found by finding the product of area of base and its height. Can we find the volume of a cylinder in the same way? Just like cuboid, cylinder has got a top and a base which are congruent and parallel to each other. Its lateral surface is also perpendicular to the base, just like cuboid. So the

Volume of a cuboid = area of base × height = l × b × h

= lbh Volume of cylinder = area of base × height

= $\pi r^2 \times h = \pi r^2 h$



Volume and Capacity:

There is not much difference between these two words.

(a) Volume refers to the amount of space occupied by an object.

(b) Capacity refers to the quantity that a container holds.

Note: If a water tin holds 100 cm^3 of water then the capacity of the water tin is 100 cm^3 . Capacity is also measured in terms of litres. The relation between litre and cm^3 is, 1 mL = 1 cm^3 ,1 L = 1000 cm^3 . Thus, 1 m^3 = 1000000 cm^3 = 1000 L.

Important Points:

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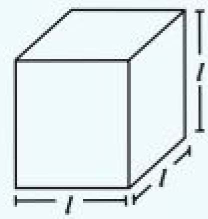
1. Area of

(i) a trapezium = half of the sum of the lengths of parallel sides × perpendicular distance between them.

(ii) a rhombus = half the product of its diagonals.

2. **Surface area** of a solid is the sum of the areas of its faces.



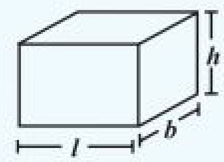
3. Surface area of

a cuboid = $2(lb + bh + hl)$

a cube = $6l^2$

a cylinder = $2\pi r(r + h)$

4. Amount of region occupied by a solid is called its **volume**.

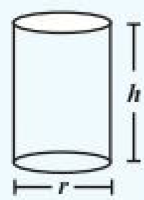


5. Volume of

a cuboid = $l \times b \times h$

a cube = l^3

a cylinder = $\pi r^2 h$



6. (i) $1\ \text{cm}^3 = 1\ \text{mL}$

(ii) $1\text{L} = 1000\ \text{cm}^3$

(iii) $1\ \text{m}^3 = 1000000\ \text{cm}^3 = 1000\text{L}$

FUNDAMENTAL CONCEPTS

Results on Triangles:

i. Sum of the angles of a triangle is 180°.

ii. The sum of any two sides of a triangle is greater than the third side.

iii. **Pythagoras Theorem:**
In a right-angled triangle, $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$.

iv. The line joining the mid-point of a side of a triangle to the positive vertex is called the **median**.

v. The point where the three medians of a triangle meet, is called **centroid.** The centroid divided each of the medians in the ratio 2 : 1.

vi. In an isosceles triangle, the altitude from the vertex bisects the base.

vii. The median of a triangle divides it into two triangles of the same area.

viii. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

Results on Quadrilaterals:

ix. The diagonals of a parallelogram bisect each other.

x. Each diagonal of a parallelogram divides it into triangles of the same area.

xi. The diagonals of a rectangle are equal and bisect each other.

xii. The diagonals of a square are equal and bisect each other at right angles.

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xiii. The diagonals of a rhombus are unequal and bisect each other at right angles.
xiv. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
xv. Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE:

I. 1. Area of a rectangle = (Length x Breadth).

$\text{Length} = \frac{\text{Area}}{\text{Breadth}}$ and $\text{Breadth} = \frac{\text{Area}}{\text{Length}}$.

II. 2. Perimeter of a rectangle = 2(Length + Breadth).

III. Area of a square = $(\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

IV. Area of 4 walls of a room = 2 (Length + Breadth) x Height.

V. 1. Area of a triangle = $\frac{1}{2}$ x Base x Height.

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a + b + c)$.

3. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Radius of incircle of an equilateral triangle of side a = $\frac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side a = $\frac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi-perimeter r = $\frac{\Delta}{s}$.

VI. 1. Area of parallelogram = (Base x Height).

2. Area of a rhombus = $\frac{1}{2}$ x (Product of diagonals).

3. Area of a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x distance between them.

VII. 1. Area of a circle = πR^2, where R is the radius.

2. Circumference of a circle = $2\pi R$.

3. Length of an arc = $\frac{2\pi R\theta}{360}$, where θ is the central angle.

4. Area of a $\frac{1}{}$ (arc x = πR^2.

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sector = $\frac{}{2}$ R) $\frac{\theta}{360}$

VIII. 1. Circumference of a semi-circle = πR.

2. Area of semi-circle = $\frac{\pi R^2}{2}$.