

## Topic: Linear Equations

### Introduction:

An equation whose solution is a straight line. In a linear equation, the variables are raised to the first power there are no variables in denominators, no variables to any power (other than one), and no variables under root signs.

For Example  $2x + 4 = 0$  --- eq (1)

Solving linear equations means finding out the unknown (usually only one but possibly several). In the above equation  $x$  is the unknown but there can be more than 2 or more variables (unknown) in a linear equations as given below.

$2x + 3y = 12$  --- eq (2)

$6x + 8y + 9z = 12$  --- eq (3)

So, A linear equation is an equation that can be written in the form

$$y = ax + b$$

where  $x$  and  $y$  are variables &  $a$  and  $b$  are constants

Note that the exponent on the variable of a linear equation is always 1.

These are examples of linear expressions:

$$x + 4$$

$$2x + 4$$

$$2x + 4y$$

To solve linear equations we will make heavy use of the following facts.

1. If  $a = b$   $a = b$  then  $a + c = b + c$   $a + c = b + c$  for any  $c$ . All this is saying is that we can add a number,  $c$ , to both sides of the equation and not change the equation.
2. If  $a = b$   $a = b$  then  $a - c = b - c$   $a - c = b - c$  for any  $c$ . As with the last property we can subtract a number,  $c$ , from both sides of an equation.
3. If  $a = b$   $a = b$  then  $ac = bc$   $ac = bc$  for any  $c$ . Like addition and subtraction we can multiply both sides of an equation by a number,  $c$ , without changing the equation.

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4. If  $a = b$  then  $\frac{a}{c} = \frac{b}{c}$  for any non-zero  $c$ . We can divide both sides of an equation by a non-zero number,  $c$ , without changing the equation.

These facts form the basis of almost all the solving techniques that we'll be looking at in this chapter so it's very important that you know them and don't forget about them. One way to think of these rules is the following. What we do to one side of an equation we have to do to the other side of the equation. If you remember that then you will always get these facts correct.

Following equations are not linear expressions:

- $x^2$  (no exponents on variables)
- $2xy + 4$  (can't multiply two variables)
- $2x / 4y$  (can't divide two variables)
- $\hat{x}$  (no square root sign on variables)

In equation 1 if we put  $x = -2$  then it satisfies the condition i.e  $2(-2) + 4 = 0$ , So 2 is the solution for eq(1)

Like so the values of  $(x, y)$  that satisfies eq(2) are  $(3, 2)$  Now let's solve one equation step by step

### Process for Solving Linear Equations:

1. If the equation contains any fractions use the least common denominator to clear the fractions. We will do this by multiplying both sides of the equation by the LCD. Also, if there are variables in the denominators of the fractions identify values of the variable which will give division by zero as we will need to avoid these values in our solution.
2. Simplify both sides of the equation. This means clearing out any parenthesis, and combining like terms.
3. Use the first two facts above to get all terms with the variable in them on one side of the equations (combining into a single term of course) and all constants on the other side.
4. If the coefficient of the variable is not a one use the third or fourth fact above (this will depend on just what the number is) to make the coefficient a one.

Note that we usually just divide both sides of the equation by the coefficient if it is an

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integer or multiply both sides of the equation by the reciprocal of the coefficient if it is a fraction.

5. **VERIFY YOUR ANSWER!** This is the final step and the most often skipped step, yet it is probably the most important step in the process. With this step you can know whether or not you got the correct answer long before your instructor ever looks at it. We verify the answer by plugging the results from the previous steps into the original equation. It is very important to plug into the original equation since you may have made a mistake in the very first step that led you to an incorrect answer.

Also, if there were fractions in the problem and there were values of the variable that give division by zero (recall the first step...) it is important to make sure that one of these values didn't end up in the solution set. It is possible, as we'll see in an example, to have these values show up in the solution set.

### Example :

Find the for y:  $7y + 5 - 3y + 1 = 2y + 2$

First combine the similar terms on the left side.

We'll start with  $7y$  and  $-3y$ .

(Don't forget to take the sign in front of the term. If there isn't a sign in front of the term, it is considered +.)

$7y - 3y = 4y$ . So we have:  $4y+6=2y+2$

$\Rightarrow 4y - 2y = 2 - 6$

$\Rightarrow 2y = -4$  or  $y = -2$

Linear inequalities are solved much the same way as linear equations with one exception: when multiplying or dividing both sides of an inequality by a negative number the inequality sign must be reversed For example  $2 < 3$  but  $-2 > -3$ . Adding and subtracting the same quantity to both sides of an inequality never changes the direction of the inequality sign.

### Read more:

Linear sentences in one variable may be equations or inequalities. What they have in common is that the variable has an exponent of 1, which is understood and so never written (except for teaching purposes). They also can be represented on a graph in the form of a straight line.

An **equation** is a statement that says two mathematical expressions are equal. A **linear equation** in one variable is an equation with the exponent 1 on the variable. These are

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also known as **first-degree equations**, because the highest exponent on the variable is 1. All linear equations eventually can be written in the form  $ax + b = c$ , where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ . It is assumed that you are familiar with the addition and multiplication properties of equations.

- *Addition property of equations:* If  $a$ ,  $b$ , and  $c$  are real numbers and  $a = b$ , then  $a + c = b + c$ .
- *Multiplication property of equations:* If  $a$ ,  $b$ , and  $c$  are real numbers and  $a = b$ , then  $ac = bc$ .

The goal in solving linear equations is to isolate the variable on either side of the equation by using the addition property of equations and then to use the multiplication property of equations to change the coefficient of the variable to 1

### Example 1:

Solve for  $x$ .

$$6(2x - 5) = 4(8x + 7).$$

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$$12x - 30 = 32x + 28$$

To isolate the  $x$ 's on either side of the equation, you can either add  $-12x$  to both sides or add  $-32x$  to both sides.

$$12x - 30 = 32x + 28$$

$$\begin{array}{r} \underline{-12x} \quad \underline{-12x} \\ -30 = 20x + 28 \end{array}$$

Isolate the  $20x$ .

$$\begin{array}{r} \underline{-28} \quad \underline{-28} \\ -58 = 20x \end{array}$$

Multiply each side by  $\frac{1}{20}$  (or divide each side by 20).

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$$\frac{1}{20}(-58) = \frac{1}{20}(20x)$$

$$-\frac{29}{10} = x$$

The solution is  $-\frac{29}{10}$ . This is indicated by placing the solution inside braces to form a

$$\text{set } \left\{ -\frac{29}{10} \right\}.$$

This set is called the **solution set** of the equation. You can check this

solution by replacing  $x$  with  $-\frac{29}{10}$  in the original equation. The solution set is  $\left\{ -\frac{29}{10} \right\}$ .

### Example 2

Solve for  $x$ .

$$\frac{2x-5}{4} - \frac{x}{3} = 2 - \frac{x+4}{6}$$

This equation will be made simpler to solve by first clearing fraction values. To do this, find the least common denominator (LCD) for all the denominators in the equation and multiply both sides of the equation by this value, using the distributive property.

$$\frac{2x-5}{4} - \frac{x}{3} = 2 - \frac{x+4}{6}$$

$$\text{LCD} = 12$$

$$12 \left( \frac{2x-5}{4} - \frac{x}{3} \right) = 12 \left( 2 - \frac{x+4}{6} \right)$$

$$\cancel{12}^3 \left( \frac{2x-5}{\cancel{4}_1} \right) - \cancel{12}^4 \left( \frac{x}{\cancel{3}_1} \right) = 12(2) - \cancel{12}^2 \left( \frac{x+4}{\cancel{6}_1} \right)$$

$$6x - 15 - 4x = 24 - 2x - 8$$

Don't forget that the  $-2$  is distributed over *both* the  $x$  and the  $4$ . Simplify both sides by

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combining like terms.

$$2x - 15 = 16 - 2x$$

Get the variable on one side.

$$\begin{array}{r} +2x \qquad \qquad +2x \\ 4x - 15 = 16 \end{array}$$

$$4x - 15 = 16$$

Isolate the variable.

$$\begin{array}{r} +15 \quad +15 \\ 4x \qquad = 31 \end{array}$$

$$4x = 31$$

Multiply each side by  $\frac{1}{4}$

$$\frac{1}{4}(4x) = \frac{1}{4}(31)$$

(or divide each side by 4).

$$x = \frac{31}{4}$$

You can check this for yourself. The solution set is  $\left\{ \frac{31}{4} \right\}$ .

For CAT you should be solving these equations in seconds with out requiring pen and paper

1.  $x - 4 = 10$  ans 14

2.  $2x - 4 = 10$  ans 7

3.  $5x - 6 = 3x - 8$  ans 7

4.  $2(3x - 7) + 4(3x + 2) = 6(5x + 9) + 3$

5. If  $0.16x + 1.1 = 0.2x + 0.95$ , then  $x = ?$  Ans

Now let's move to linear equation involving more than one variable , for ex.

$$3x + 2y = 12 \text{ and } 3x + 5y = 18$$

Above two equations are quite easy to solve, one of the variable  $x$  has same co-efficient in both the lines, so we can easily make it as  $5y - 2y = 18 - 12$  or  $3y = 6$  or  $y = 2$  and thus  $x = 8/3$

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Now if there is no variable with same co-efficient then we have to make it by multiplying with a factor.

$$4x + 7y = 15 \text{ -----eq(4)}$$

$$2x + 3y = 11 \text{ -----eq(5)}$$

Above two equations can be written as

$$8x + 14y = 30 \text{ -----eq(6)}$$

$$8x + 12y = 44 \text{ -----eq(7)}$$
 We have multiplied eq(4) with 2 and eq(5) with 4

Now eq(6) and (7) can be easily solved but the big question is how to know what to multiply? (left for you to figure out).

In CAT you can't expect to get any direct questions from this section but If you practice a lot in solving linear equations without using pen & paper then it will help in solving other questions quickly.