Topic: Geometry

Introduction:

Geometry in CAT is one of the easiest sections of QA, if you spend enough time preparing for it, you can easily secure 30%, so get started right away!

ZERO DIMENSION / NO DIMENSION:

POINT : A point is a mark of position which has no dimension, i.e. no shape or size .

Note-1 : Infinite lines can be drawn through a points.

Note -2 : One and only one line can be drawn through two distinct point.

Note – 3 : Three or more than three points said to be collinear. if a line segment passing through them, Otherwise they are called Non-collinear.

Note- 4 : Four or more than four points said to be concyclic .if a circle passes through them.

A, B, C & D are concylic points.

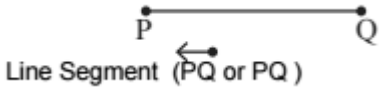
Note-5 : Any three non-collinear points are always concyclic.

Note-6 : A circle always passes through three non-collinear points.

ONE DIMENSION

LINE : A line is a straight path that extends indefinitely in both the directions. It has no end points.

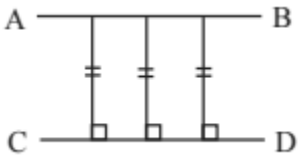
LINE SEGMENT: A line segment is the portion of a line with two fixed end points.



PARALLEL LINES : Two lines in a plane are called parallel if they do not meet when produced indefinitely on either side

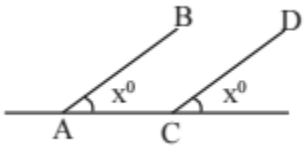
Note-1 : Perpendicular distance between two parallel lines are always equal.

Topic: Geometry



AB | | CD

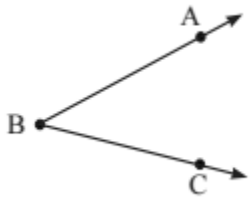
Note-2 : If two straight lines make equal angle with same plane then they are parallel and vice-versa.



AB | | CD

TWO DIMENSION:

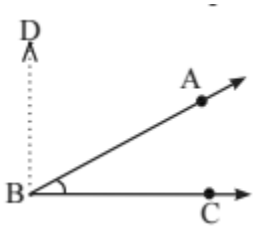
ANGLE : An angle is formed when two line segments or two rays have a common end-point. The two line segments forming an angle are called arms of the angle, whereas their common end-point is called the vertex of the angle.



Angle ABC ($\angle$ ABC)

TYPES OF ANGLES (According to measurement of angle)

1**) Acute Angle :** An angle measuring less than 900 is called an Acute Angle.



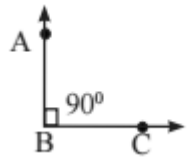
Acute Angle ABC ($\angle$ ABC < 900)

2**) Right Angle** : An angle whose measure is 900, is called a right angle. The arms of a right

Topic: Geometry

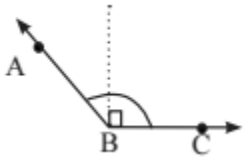
angle are

perpendicular to each others .



Right Angle ABC (∠ ABC = 900)

3) **Obtuse Angle** : An angle greater than a right angle, but less than 1800 , is called an obtuse angle.

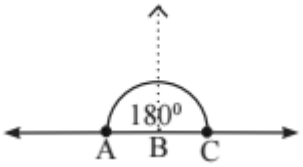


Obtuse angle ABC (900 < ∠ ABC < 1800)

4) **Straight Angle** : An angle equal to two right angles.

Or

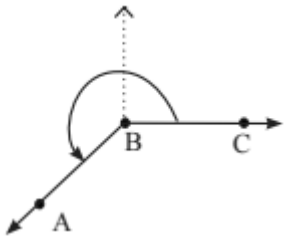
An Angle whose measure is 1800 is called a straight angle.



Straight angle ABC (∠ ABC = 1800)

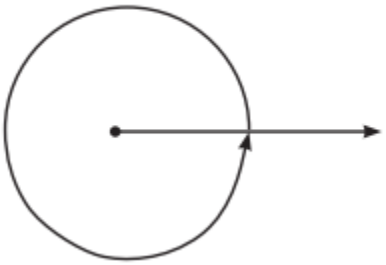
5) **Reflex Angle** : An angle whose measure is more than 1800 and less than 3600 is called a reflex angle.

Topic: Geometry



Reflex angle ABC (1800 < ∠ ABC ∠ 3600)

6) **Complete Angle** : The measure of a complete angle is 3600.



TERMS RELATED TO ANGLE:

1) **Complementary Angles:** Two angles are said to be complementary if the sum of their degree measure is 900 .

2) **Supplementary Angles** : Two angels are said to be supplementary if the sum of their degree measures is 1800 .

3) **Adjacent Angles** : Two angles are said to be adjacent if they have a common vertex and a common arm between two other arms.

4) **Linear Pair Angles** : A pair of adjacent angles is said to form a linear pair if the outer arms of the angles lie on one line. A linear pair (consisting of two angles) is measured to be 1800

5) **Vertically Opposite Angles** : The angles opposite to the common vertex formed by the intersection of two lines having no common arm are known as vertically opposite angles.

Note : When two lines intersect, vertically opposite angles are always equal.

6) **Perpendicular** : The two lines are said to be perpendicular to each other, if they contain an angle of 900 or one right angle between them.

7) **Perpendicular Bisector** : If a line passes through the mid-point of a line segment and perpendicular to it, then the line is called the perpendicular bisector of the line segment.

Topic: Geometry

Note : Every point on perpendicular bisector is equidistant from both ends.

8) **Angle Bisector** : If a line bisects an angle, then the line is called the bisector of the angle.

Note : Every point on angle bisector is equidistant from both arms.

MEASUREMENT OF ANGLE :

There are three system used for the measurement of angles.

1) Sexagesimal System or English System (Degree)

2) Centesimal System or French System (Grade)

3) Circular measurement (radian)

1) **Sexagesimal or English System (Degree)** : Here a right angle is divided into 900 equal parts known as degrees. Each degree is divided into 60 equal parts called minutes and each minutes is further divided into 60 equal parts called seconds.

60 seconds (60") = 1 minute (1')

60 minutes (60') = 1 degree (1^0)

90 degree (90^0) = 1 right angle.

2) **Centesimal system or French System (Grade)** : Here a right angle is divided into 100 equal parts called grades and each grade is divided into 100 equal parts, called minute and each minute is further divided into 100 seconds.

100 seconds (100") = 1 minute (1')

100 minutes (100') = 1 grade (1g)

100 grade (1000) = 1 right angle

3) **Circular measurement or Radian measure** : The number of radians in an angle subtended by an arc of a circle at the centre is equal to arc.

The angle subtended by arc of length radius at the centre is equal to one radian.

Angle made by Needle of a Clock

Hours Needle –

1 dial = 360^0

Topic: Geometry

12 hours = 360^0

1 hours = 30^0

60 minute = 30^0

1 minute = $1^\circ / 2$

Minutes Needle

1 dial = 360^0

60 minute = 360^0

1 minute = 6^0

POLYGON :

Polygon : A plane geometrical figure, bounded by atleast three line segments, is called a polygon.

Name of Polygons :

No. of Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Types of Polygons :

i) **Convex Polygon** : If each angle of a polygon is less than 180^0 , it is called a convex polygon.

ii) **Concave Polygon** : If at least one angle of a polygon is more than 180^0 , it is called a concave polygon.

iii) **Regular Polygon** : A regular polygon is a polygon with all its sides and all its angles equal.

iv) **Non-Regular Polygon** : A polygon is called a non-regular polygon, if all the sides are not equal.

Topic: Geometry

Terms related to Polygon :

Diagonal : Line segment joining any two nonconsecutive vertices.

FORMULA related to Polygon

1) Sum of interior angles of a polygon = $(n - 2) \times 180^0$

2) Sum of exterior angles = 360^0

3) Number of diagonals

$$= \frac{n(n-3)}{2} = {}^{n}C_2 - n$$

4) Sum of vertices angles of n sided star shaped polygon = $(n - 4) \times 180^0$

5) Interior angle + Exterior angle = 180^0

6) Exterior angle = 180^0 – Interior angle

Properties related to Polygon :

1. In any polygon (except triangle and quadrilateral) sum of interior angles is greater than sum of exterior angles.

2. Triangle is only one polygon in which sum of interior angles is half of sum of exterior angles.

3. Quadrilateral is only one polygon in which sum of interior angles is equal to sum of exterior angles.

TRIANGLE :

Triangle - A triangle is a plane and closed geometrical figure, bounded by three line segments. A triangle has three sides , three angles and three vertices –

Types of triangle (According to side)

1) **Equilateral Triangle** : A triangle in which all the three sides are equal.

Note (1) : In equilateral triangle all angles are equal.

Note (2) : In equilateral triangle each angle is equal to 600 .

2) **Isosceles Triangle** : A triangle in which any of two sides are equal.

Topic: Geometry

Note (3) : In Isosceles triangle two angles are equal.

Note (4) : If two sides of a triangle are equal than angle opposite to them are equal.

Note (5) : If two angles of a triangle are equal than sides opposite to them are equal. 3) Scalene Triangle: A triangle in which all the sides are unequal.

Note (6) : In scalene triangle all the three angles are unequal.

Note (7) : If two sides of a triangles are unequal than greater side has greater angle opposite to it.

Note (8) : If two angles of a triangle are unequal than greater angle has greater side opposite to it.

Types of Triangle (According to angle)

1) **Acute-angled Triangle**: If all the three angles of a triangle are acute angles it is called an acute-angled triangle.

Note (9) : In acute angle triangle sum of any two angles is greater than 90^0 .

Note (10) : In acute angle triangle $c^2 < a^2 + b^2$ (where a, b & c are length of sides and C is greatest side.

2) **Right-angled Triangle** : If one of the angles of a triangle is a right angle, it is called right-angled triangle.

Note (11) : In right angle triangle sum of other two angle is equal to 90^0 .

Note (12) : If sum of two angles is equal to third angle than triangle is right angled triangle.

Note (13) : In right angled triangle $c^2 = a^2 + b^2$ where a, b & c are length of sides and c is greatest side.

3) **Obtuse-angled Triangle** : If one of the angles of a triangle is an obtuse angle, it is called an obtuse-angled triangle.

Note (14) : If sum of two angle is less than 90^0 , than triangle is obtuse angled triangle.

Note (15) : In Obtuse angled triangle $c^2 > a^2 + b^2$ where a, b & c are length of sides and c is greatest side.

Terms related to Triangle:

1) **Median** : The straight line joining a vertex of a triangle to the mid-point of the opposite side is

Topic: Geometry

called a median. A triangle has three medians.

Note (16) : In equilateral triangle all the three medians are equal in length.

Note (17) : In Isosceles triangle medians drawn from vertex of each equal angles are equal. That means in Isosceles triangle two medians are equal.

Note (18) : In isosceles triangle median drawn from vertex of unequal angle is perpendicular to side.

Note (19) : In Isosceles triangle median drawn from vertex unequal angle is bisect the vertex angle.

Note (20) : In scalene triangle all the three medians are unequal.

Note (21) : In any triangle median lie always inside of triangle.

Note (22) : The three medians of a triangle are concurrent. That means they have a common point of intersection.

Note (23) : In right-angle triangle median drawn from vertex of right angle to hypotenuse is equal to half of the hypotenuse.

Or, If median is equal to half of its corresponding side than triangle must be right-angled triangle.

2) **Centroid** : The three medians of a triangle always intersect each-other at the same point. This point of intersection of the medians is called centroid of the triangle.

Note (24) : Centroid divides the median into the ratio of 2 : 1.

3) **Altitude / Perpendicular / Height** : An altitude of a triangle, with respect to a side, is the perpendicular line segment drawn to the side from the opposite vertex.

Note (25) : In equilateral triangle all the three perpendicular are equal in length.

Note (26) : In equilateral triangle perpendicular and median are same line segment.

Note (27) : In Isosceles triangle two perpendicular drawn from equal angles to equal sides are equal and perpendicular drawn from unequal angle to unequal side is also median and also angle bisector.

Note (28) : In scalene triangle all the three perpendicular are unequal.

Note (29) : In acute angle triangle all the three perpendicular lie inside of triangle.

Note (30) : In right angle triangle two sides containing right angles are also altitude and one altitude from vertex of right angle is inside of triangle.

Topic: Geometry

Note (31) : In obtuse angle triangle two altitudes from acute angles are outside of triangle and one altitude is inside of triangle.

Note (32) : Greatest side has least altitude and least side has greatest altitude.

Note (33) : In any line segment joining vertex to opposite side perpendicular is shortest.

Note (34) : The three perpendiculars are concurrent.

4) **Orthocentre** : The three altitudes of a triangle always intersect each-other at the same point. This point of intersection of the altitudes is called orthocenter of the triangle

Note (35) : Angle made by any side on ortho-centre is supplementary of opposite angle.

Note (36) : In equilateral triangle centroid and orthocentre are same point. Note (37) : In isosceles triangle centroid and orthocenter are two different points lie on the perpendicular or median drawn from unequal angle to unequal side.

Note (38) : In isosceles triangle vertex centroid and orthocenter are collinear points.

Note (39) : A line segment joining centroid and orthocenter makes 900 with side or bisect the side then triangle is isosceles triangle.

Note (40) : In scalene triangle vertex, centroid and orthocenter are three non-collinear points.

Note (41) : In acute angle triangle orthocenter lie inside of triangle.

Note (42) : In right angle triangle orthocenter is vertex of right angle.

Note (43) : In obtuse angle triangle orthocentre lie outside of triangle.

5) **Angle Bisector** : A line segment joining vertex to opposite side such that it bisect the vertex angle.

Note (44) : In equilateral triangle all the three angle bisectors are equal in length.

Note (45) : In equilateral triangle angle bisector, perpendicular and median are same line segment.

Note (46) : In isosceles triangle angle bisectors drawn from equal angles are equal in length and angle bisector drawn from unequal angle is also perpendicular and median

Note (47) : In scalene triangle all the three angle bisectors are unequal in length. Note (48) : Angle bisector lie always inside of Δ.

Note (49) : All the three angle-bisectors are concurrent.

Topic: Geometry

6) **Incentre** : The point of intersection of the internal bisectors of the angles of a triangle is called its incentre.

7) **Incircle**: A circle inside of triangle touches all the three sides of triangle and its centre is incentre of triangle.

8) **Perpendicular Bisector**: If a line passes through the mid-point of a side of a triangle and perpendicular to it, then the line is called the perpendicular bisector of the line segment.

9) **Circumcentre**: The point of intersection of the perpendicular bisectors of the sides of a triangle is called circumcentre.

10) **Circumcircle** : Circumcircle is a circle passing through all the three vertices of triangle and its centre is circumcentre of triangle .

Congruence of Triangle :

Two triangle are said to be congruent if they are equal in shape and size both.

Or,

Two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.

SUFFICIENT CONDITIONS (CRITERIA) FOR CONGRUENCE OF TRIANGLES

1) **SIDE – SIDE – SIDE (S-S-S) Congruence Criterion** : Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

2) **SIDE-ANGLE-SIDE (S-A-S) Congruence Criterion** : Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle.

3) **ANGLE-SIDE-ANGLE (A-S-A) Congruence Criterion** : Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

4) **ANGLE-ANGLE-SIDE (A-A-S) Criterion of Congruence** : If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then two triangles are congruent.

5) **RIGHT ANGLE-HYPOTENUSE-SIDE (R-H-S) Congruence Criterion** : Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.

PROPERTIES RELATED TO CONGRUENCE OF TRIANGLES :

Topic: Geometry

Note-1 : If two triangles are congruent then their corresponding sides are equal.

Note-2 : If two triangles are congruent then their corresponding angles are equal.

Note-3 : If two triangles are congruent then they must be equiangular but if two triangles are equiangular then they need not be congruent.

Note-4 : If two triangles are congruent then they are equal in area and perimeter.

Note-5 : If two triangles are congruent then their all corresponding parts are equal.

SIMILARITY OF TRIANGLES :

Two triangle are said to be similar if they are equal in shape but need not to be equal in size.

Or,

Two triangles said to be similar if their corresponding angles are equal and their corresponding sides are proportional.

CRITERIA OF SIMILARITY :

1) **A – A / A – A – A** : If two triangles are equiangular, then they are similar.

2) **S – S – S** : If the corresponding sides of two triangles are proportional, then they are similar.

3) **S – A – S** : If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

PROPERTIES RELATED TO TRIANGLE :

1) The sum of the three angles of a triangle is 180^0 .

2) Sum of any two sides of a triangle is greater than the third side.
3) Difference between any two sides of a triangle is less than the third side.

4) If a side of triangle is produced the exterior angle so formed is equal to the sum of the two interior opposite angles.

5) Angle made by bisector of any two angles is equal to 90^0+1/2 of third angle.

6) Angle made by bisectors of any two exterior angles is equal to 90^0–1/2 of third angle.

7) Angle made by bisectors of one interior and one exterior angle is equal to half of third angle.

8) Angle made by perpendicular and angle bisector on vertex is equal to half of difference of other two angles.

Topic: Geometry

9) Sum of exterior and interior opposite angle is equal to twice of the angle made by angle bisector on the same side.

10) Sum of any two sides greater than twice the median drawn to the third side.

FORMULA RELATED TO TRIANGLE

1) Area of triangle = $\frac{1}{2}$ x base x height

2) Area of triangle = $\sqrt{s(s-a)(s-b)(s-c}$

Where, s = $\frac{a+b+c}{2}$ and a, b & c are length of sides.

3) Area of triangle = $\frac{4}{3}\sqrt{s(s-a)(s-b)(s-c}$

Where, s = $\frac{a+b+c}{2}$ and a, b & c are length of medians.

4) Height of equilateral triangle = $\frac{\sqrt{3}}{2} \times \text{side}$

5) Length of side of equilateral triangle

= $\frac{2}{\sqrt{3}} \times \text{height}$

Topic: Geometry

6) Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times \text{side}^2$

7) Area of equilateral triangle = $\frac{(\text{height})^2}{\sqrt{3}}$

8) Height of isosceles triangle = $\frac{1}{2}\sqrt{4b^2 - a^2}$
Where b is equal side

9) Area of isosceles triangle = $\frac{a}{4}\sqrt{4b^2 - a^2}$

QUADRILATERAL : A geometrical figure bounded by four line segment is called quadrilateral.

Properties related to quadrilateral

1) Sum of interior angles is equal to 360^0

2) Sum of exterior angles is equal to 360^0

3) Angle made by bisectors of any two consecutive angles is equal to half of sum of others to angles.

4) Sum of pair of interior opposite angle is equal to sum of pair of other two exterior opposite angles.

5) Line segment joining midpoints of any two adjacent sides is parallel and equal to half of corresponding diagonal.

6) Quadrilateral formed by line segment joining midpoints of sides of original quadrilateral is a parallelogram.

7) Area of quadrilateral joining midpoints of sides is equal to half of original quadrilateral.

TYPES OF QUADRILATERAL :

1) **Parallelogram**: A quadrilateral whose both pair of opposite sides are parallel, is called a parallelogram.

2) **Rectangle** : Rectangle is a parallelogram in which each angle is equal to 90^0 .

3) **Square**: Square is a parallelogram in which all sides are equal and each angle is equal to 90^0 .

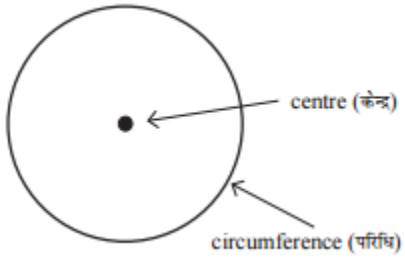
Topic: Geometry

4) **Rhombus** : Rhombus is a parallelogram in which all sides are equal.

5) **Trapezium**: A quadrilateral in which one pair of opposite sides are parallel.

CIRCLE :

A circle is a simple closed curve, all the points of which are at the same distance from a given fixed point. The fixed point is called centre of the circle.



1) **Radius** : Line segment joining centre and any point of circle.

2) **Chord** : A line segment joining any two points on a circle is called chord of the circle.

3) **Diameter**: A chord passing through the centre of a circle is known as its diameter. Diameter is longest chord.

4) **Arc of a circle** : A continuous piece of a circle is called an arc.

5) **Concentric Circles** : Circles having the same centre are said to be concentric circles.

6) **Secant of a circle** : A straight line intersecting the circle at two points, is called a secant.

7) **Tangent of a circle** : A straight line touching the circle at one point only is called a tangent.

8) **Semicircle** : A diameter of a circle divides the circumference of the circle into two equal arcs and each of these arcs is known as a semicircle.

9) **Sector of a circle** : The part of a circle enclosed by an arc and two radii is called a sector.

10) **Segment of a circle** : The part of the circular region enclosed by an arc and the chord joining the end points of the arc is called a segment of the circle.

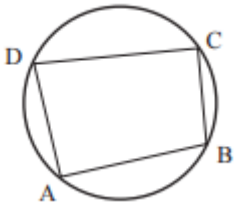
11) **Central Angle** : An angle subtended by an arc (or a chord) at the centre is called a central angle.

12) **Inscribed Angle** : An angle, whose vertex lies on the circumference of a circle and the two arms are the chords of the circle, is called an inscribed angle.

Topic: Geometry

Cyclic Properties of Circle:

Cyclic quadrilateral : A cyclic quadrilateral is called cyclic quadrilateral if its all vertices lie on a



circle.

1. The sum of either pair or opposite angles of a cyclic quadrilateral is 180^0 .

2. If the sum of any pair of opposite angles of a quadrilateral is 180^0 , then the quadrilateral is cyclic.

3. If one side of a cyclic quadrilateral is produced, then the exterior angle is equal to interior opposite angle.

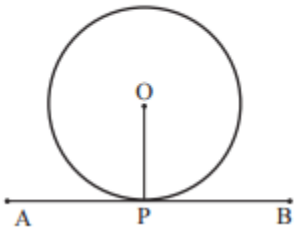
4. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

5. If two sides of a cyclic quadrilateral are parallel then the remaining two sides are equal and diagonal are also equal.

6. If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.

7.The sum of the angles in the four segments exterior to a cyclic quadrilateral is equal to 6 right angles.

A tangent to a circle is perpendicular to the radius through the point of contact.



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Topic: Geometry

Area and Perimetre of Circle

1. Area of circle = πr^2
2. Perimetre of circle = $2\pi r$
3. Area of semicircle = $\frac{1}{2}\pi r^2$
4. Perimeter of semicircle = $(\pi+2)r$
5. Area of a quadrant of a circle = $\frac{1}{4}\pi r^2$
6. Perimeter of a quadrant of a circle = $\left(\frac{\pi}{2}+2\right)r$
7. Area of a sector of a circle = $\frac{\theta}{360^\circ}\times\pi r^2$
8. Length of arc = $\frac{\theta}{360^\circ}\times 2\pi r$