# Topic: Averages

## Introduction:

Average is a tool which is extensively used in data analysis. It is a relative value which lies near the original values. But when it is discussed with respect to consecutive observations it turn into their mean value or mid-value.

1. Average:

Average = (Sum of observations / Number of observations)

2. Average Speed:

Suppose a man covers a certain distance at x kmph and an equal distance at y kmph.

Then, the average speed druing the whole journey is (2xy/ x + y) kmph.

## What is Average?

- A calculated "central" value of a set of numbers.
- To calculate: add up all the numbers, then divide by how many numbers there are.
- Example: what is the average of 2, 7 and 9?
- Add the numbers: 2 + 7 + 9 = 18
- Divide by how many numbers (i.e. we added 3 numbers): 18 ÷ 3 = 6
- So the average is 6

## Read more:

- Average= $(x_1+x_2+x_3+.....x_n)/n$
- where $x_1,x_2,x_3$, are quantities.
- Weighted Average= $(n_1x_1+n_2x_2+n_3x_3+.....n_kx_k) / n_1+n_2+n_3+.....n_k$
  where $x_1,x_2,x_3.... x_k$ are the quality factors
  $n_1,n_2,n_3,.....n_k$ are the quantity factors

  Eg: If the average height of boys= 172cms and that of girls=154 cms, then find average height of the class with 18 boys and 12 girls?
- Here $n_1$ and $n_2$ are no. of boys and girls (Quantity factor)
  $x_1$ and $x_2$ are average heights (Quality factor)

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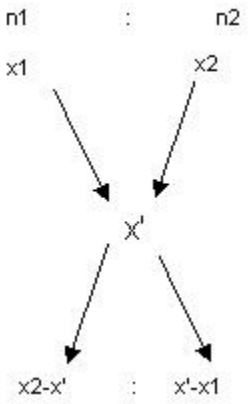
- **Mixtures**
  For mixtures, Average, $x' = (n_1x_1+n_2x_2)/(n_1+n_2)$
- **Alligation:**




$\Rightarrow n_1/n_2 = (x_2-x')/x'-x_1$

## Rules of Average in Details:

**Rule 1:**
In the Arithmetic Progression there are two cases when the number of terms is odd and second one is when number of terms is even.
So when the number of terms is odd the average will be the middle term.
And when the number of terms is even then the average will be the average of two middle terms.

**Examples 1**: what will be the average of 13, 14, 15, 16, 17?
**Solution**: Average is the middle term when the number of terms is odd, but before that let's checks whether it is in A.P or not, since the common difference is same so the series is in A.P.
So the middle term is 15 which is our average of the series.

**Let's check it in another way**.
In the first statement of the article we have written that the average of a set of terms is equal to:
Sum of all terms / Number of terms
So the sum of all terms in this case is 75 and the number of terms is 5 so the average is 15.

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Now come to the second form when the number of terms are even

**Example 2**: What will be the average of 13, 14, 15, 16, 17, 18?
Solution: We have discussed that when the number of terms are even then the average will be the average of two middle terms.
Now the two middle terms are 15 and 16, but before that the average we must check that the series should be A.P. Since the common difference is same for each of the term we can say that the series is in A.P.
And the average is (16+15)/2 = 15.5

**Rule 2:**
The average of the series which is in A.P. can be calculated by ½(first + last term)
**Example 1:** What will be the average of 216, 217 , 218?
Solution: So the answer would be = ½ (216 + 218) = 217
(Which is also the middle term of the series)

**Example2:**
What will be the average of first 10 natural numbers?
Solution: The first 10 natural numbers are 1,2,3,4,5,6,7,8,9,10
So the average will be ½ (1 + 10 ) = ½ (11) = 5.5

**Rule 3:**
If the average of n numbers is A and if we add x to each term then the new average will be = (A+ x).
**For example:** The average of 5 numbers is18. If 4 is added to each of the number then the average would be equal to __?
**Solution**: Old average = 18
New average will be = 4 + old average = 22
This is because each term is increased by 4 so the average would also be increased by 4 so the new average will be 22

**Rule 4:**
If the average of n numbers is A and if we multiply p with each term then the new average will be = (A x p).
**For Example**: The average of 5 numbers is 18. If 4 is multiplied to each of the number then the average would be equal to __?
**Solution**: Old average = 18
New average will be = 4 x 18= 72
There are two more operation which can also be applied on the same principle as the above, i.e. subtraction and division.
**Rule 5 :**
In some cases, if a number is included in the series of numbers then the average will change and the value of the newly added term will be = Given average + (number of new terms x increase in average).

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This value will also same as the New average + (number of previous terms x increase in average ) .

**For example:** The average age of 12 students is 40. If the age of the teacher also included then the average becomes 44. Then what will be the age of the teacher?
**Solution**: Average given = 40
Number of students = 12
Therefore the age of the teacher = 40 + (12 + 1) x 4 = 40 + 52 = 92
And this is also calculated as 44 + (12 x 4)= 92
Therefore the average age of the teacher is 92 yrs

**Alternatively**
The average of 12 = 40 that means the total number of units are 12 x 40 = 480
Now the new average is 44 and the number of terms are 13 so therefore the total number of units are = 44 x 13 = 572
So the included units would be equal to 572 – 480 = 92

**Rule 6:**
In some cases a number is excluded and one more number is added in the series of the number then the average will change by q and the value of the newly added term will be = Replaced Term + (increased in average x number of terms ).

**For example:**
The average age of 6 students is increased by 2years when one student whose age was 13 years replaced by a new boy then find the age of the new boy
**Solution**: The age of the boy will be = Age of the replaced boy +increase in average x number of terms
i.e. the age of the newly added boy = 13 + 2 x 6 = 25

**Rule 7:**
There are two more cases when the series is divided into two parts and one of the terms is either included or excluded, then the middle term can be calculated by following methods.

**Case1: When the term is excluded.**
Average(total ) + number of terms in first part x {average (total) – average (first part)} + number of terms in second part x {average (total) – average (second part)}

**Case 2: When the term is included.**
Average (total) + number of terms in first part x {average (first part) – average(total) }+ x number of terms in second part x {average (second part) – average (total)}

**For Example:** The average of 20 numbers is 12 .The averages of the first 12 is 11 and the average of next 7 numbers is 10. The last number will be?

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**Solution:**
Here in this case one number is excluded so the number would be =
Average(total ) + number of terms in first part x {average (total) – average (first part)} + number of terms in second part x {average (total) – average (second part)} **i.e. = 12 + 12 x (12-11)+(12-10) x 7 = 38.**