# **Topic:** Surds and Indices

In this article we will discuss about **Surds & Indices.** This topic are very useful and acts as the base of simplification and Algebra. It is compulsory that question will comes from this topic.

## Surds and Indices:

Let n be a positive integer and a be a real number, then:

$$a^n = \frac{a \times a \times a \times \ldots \times a}{(n\,factor)}$$

**where $a^n$ is called "$n^{th}$ power of a" or "a raised to the power n"**
where, a is called the **base** and n is called **index or exponent** of the power $a^n$.

## Laws of Indices:

- $a^m \times a^n = a^{m+n}$ where and (m, n)
- $a^m \times a^n \times a^p \ldots\ldots = a^{m+n+p+\ldots\ldots}$
- $$\frac{a^m}{a^n} = \begin{cases} a^{m-n} & if \quad m > n \\ \frac{1}{a^{n-m}} & if \quad n > m \\ 1 & if \quad m = n \end{cases}$$
- $(a^m)^n = a^{mn} = (a^n)^m$
- $$a^{m^n} = a^{m \times m \times \ldots\ldots n \; times} \neq (a^m)^n$$
- $(ab)^n = a^n b^n$

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- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
- $(-a)^n = \begin{cases} a^n, \text{when } n \text{ is even} \\ -a^n, \text{when } n \text{ is odd} \end{cases}$
- $a^{-n} = a^{(-1)n} = (a^{-1})^n = \left(\frac{1}{a}\right)^n = \frac{1}{a} \times \frac{1}{a} \times \frac{1}{a} \ldots\ldots n \text{ times}$
- $a^{p/q} = (a^{1/q})^p$ where p is a positive integer and q≠0
- If the index of a power is unit (i.e. 1) then the value of the power is equal to its base, i.e.

$a^1 = a, 0^1 = 0$

- $a^m = a^n \Rightarrow m = n$ when $a \neq 0, 1$
- $a^m = b^m \Rightarrow a = b$

**Surds:** If a is rational and n is a positive integer and $a^{1/n} = \sqrt[n]{a}$ is **irrational**, then $\sqrt[n]{a}$ is called a surds of order n or nth root of a.

- A surd which has unity as its rational factor **(i.e., a = 1) i**s called "pure surd". e.g $\sqrt[4]{3}, \sqrt{2}, \sqrt[3]{3}$ etc
- A surd which has a rational factor other than unity, the other irrational, is called "**mixed surd**". e.g $3\sqrt{5}, 2\sqrt{7}, 5\sqrt[3]{7}$

- If $\sqrt[n]{a}$ is a surd it implies , a is a rational number and $\sqrt[n]{a}$ is an irrational number.

**Quadratic Surd:**

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A surd of order 2 (i.e $\sqrt{a}$) is called a **quadratic** surd.

E.g. : $\sqrt{2} = 2^{1/2}$ is a quadratic surd but $\sqrt{4} = 4^{1/2}$ is not a quadratic surd because $\sqrt{4} = 2$ is a rational number. Therefore $\sqrt{4}$ is not a surd.

## Cubic Surd:

A surd of order 3 is called a cubic surd. e.g. $9^{1/3}$

## Important Formulae Based on Surds :

- $\sqrt[n]{a^n} = a$
- $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \; and \; \frac{k\sqrt[n]{a}}{l\sqrt[n]{b}} = \frac{k}{l}\sqrt[n]{\frac{a}{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$
- $\left(\sqrt[n]{a}\right)^m = (a)^{m/n} = \left(a^m\right)^{1/n} = \sqrt[n]{a^m}$
- $\left(\sqrt[n]{a}\right)^m = (a)^{m/n} = \left(a^m\right)^{1/n} = \sqrt[n]{a^m}$
- $\sqrt{a} \times \sqrt{a} = a$
- 
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $and \; k.\sqrt[n]{a} \times l.\sqrt[m]{b} = kl.\sqrt[n]{a} \cdot \sqrt[m]{b} = kl.\sqrt[mn]{a^m b^n}$
- $\sqrt{a^2 b} = a\sqrt{b}$
- $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
- $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$
- $(\sqrt{a} + \sqrt{b}) \times (\sqrt{a} - \sqrt{b}) = a - b$

## Similar or like Surds:

surds having same irrational factors are called similar or like surds.

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e.g.3√3, 4√3, 7√3 are similar surds.

**Unlike surds:**

Surds having no common irrational factors are called **unlike** surds.

e.g. 3√3, 7√5 are unlike surds.

**Comparison of Surds:**

If two surds are of the same order then the one whose radicand is larger is the larger surds.

7√3 > 3√3.

If two surds are of different order then:

**Question**:Which is larger $\sqrt{2}$ or $\sqrt[3]{3}$?

Sol. Given surds are of order 2 & 3 respectively whose L.C.M. is 6.

Convert each into a surd of order 6, as shown below :

$$\sqrt{2} = 2^{1/2} = 2^{\frac{1}{2}\times\frac{3}{3}} = 2^{3/6} = (2^3)^{1/6} = (8)^{1/6} = \sqrt[6]{8}$$

$$\sqrt[3]{3} = 3^{1/3} = 3^{\frac{1}{3}\times\frac{2}{2}} = 3^{2/6} = (9)^{1/6} = \sqrt[6]{9}$$

$$\sqrt[6]{9} > \sqrt[6]{8},\ so,\ \sqrt[3]{3} > \sqrt{2}$$

**Some Useful Results :**

**(1)** $if\ y = \sqrt{x + \sqrt{x + \sqrt{x + ........\infty}}}$

and $x = n(n+1)$ then $y = (n+1)$

e.g. $y = \sqrt{12 + \sqrt{12 + \sqrt{12 + ........\infty}}}$

we have $x = 12 = 3 \times 4 = n(n+1)$

$\therefore y = 4$

**(2)** $if\ y = \sqrt{x - \sqrt{x - \sqrt{x - ........\infty}}}$

and $x = n(n+1)$ then y=n

e.g. $y = \sqrt{12 - \sqrt{12 - \sqrt{12 - ........\infty}}}$

we have $x = 12 = 3 \times 4 = n(n+1)$

therefor y=3

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(3) If $x=\frac{4\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\frac{x+2\sqrt{a}}{x-2\sqrt{a}}+\frac{x+2\sqrt{a}}{x-2\sqrt{a}}=2$

(4)If $x=\frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\frac{x+\sqrt{a}}{x-\sqrt{a}}+\frac{x+\sqrt{a}}{x-\sqrt{a}}=2$