Topic: REMINDER

Supposing a number N is divided by another number “x”; if the quotient obtained is “Q” and the remainder obtained is “R”,

then the number can be expressed as N=Qx+R

For example, suppose 8 is divided by 3

In this case, N=8, x=3. 3×2=6, which is 2 less than 8. hence Q=2 and R=(8-6)=2 Hence 8=2×3+2

Basic Remainder Theorem

Basic remainder theorem is based on product of individual remainders.

If R is the remainder of an expression(p*q*r)/X, and pR, qR and rR are the remainders when p,q and r are respectively divided by X,

then it can be said that ((p_R x q_R x r_R))/X, will give the same remainder as given by (p*q*r)/X

Let us understand this with the help of an example

Find the remainder when (361*363) is divided by 12.

Steps

1) Take the product of individual remainders, i.e. 361/12|R =1 and 363/12|R= 3

2) Find the remainder when you divide that product by the number (361*363)/12|R= (1*3)/12|R. answer= 3

This is Basic Remainder theorem put across in Numbers

Find the remainder when 10^6 is divided by 7 i.e. $(10^6/7)_R$.

Solution:

$10^6=10^3x10^3$

Thus$(10^6/7)$R = $(10^3/7$ x $10^3/7)$R = ((6 * 6)/7)R = (36/7)R = 1.

So the remainder is 1

Concept of negative remainder

The remainder obtained on division of a number N by a divisor X can be expressed in two ways as “R” and “X-R”

For example 10/11 remainder is +10 itself. It can also be written as 10-11= -1 Similarly, 32/10 remainder is +2 or -8

Topic: REMINDER

Let us express the solution for questions 31 above, in another way- based on the concept of negative remainder Thus$(10^6/7)R = (10^3/7 \times 10^3/7)R = ((-1 * -1)/7)R = (1/7)R = 1$.

Remainder when the product of some numbers is divided by the requisite number is the product of individual remainders of the numbers-

This is Basic Remainder Theorem put across in words

Let us see why this happens

If the numbers N1,N2,N3 give remainders of R1,R2,R3 with quotients Q1,Q2,Q3 when divided by a common divisor D

N1=DQ1+R1 N2=DQ2+R2 N3=DQ3+R3

Multiplying=N1xN2xN3

=(DQ1+R1)x (DQ2+R2)x(DQ3+R3) = D(some number)+(R1xR2xR3) =first part is divisible by D,

hence you need to check for the individual remainders only

ALL POSSIBLE TYPES OF QUESTION ON REMINDER:

1. **What is the remainder when the product 1998 × 1999 × 2000 is divided by 7?**

Answer: the remainders when 1998, 1999, and 2000 are divided by 7 are 3, 4, and 5 respectively. Hence the final remainder is the remainder when the product 3 × 4 × 5 = 60 is divided by 7.

Answer = 4

2. **What is the remainder when 2^{2004} is divided by 7?**

2^{2004} is again a product (2 × 2 × 2... (2004 times)). Since 2 is a number less than 7 we try to convert the product into product of numbers higher than 7. Notice that 8 = 2 × 2 × 2. Therefore we convert the product in the following manner-

$2^{2004} = 8^{668}$ = 8 × 8 × 8... (668 times).

The remainder when 8 is divided by 7 is 1.

Hence the remainder when 8^{668} is divided by 7 is the remainder obtained when the product 1 × 1 × 1... is divided by 7

Answer = 1

3. **What is the remainder when 2^{2006} is divided by 7?**

This problem is like the previous one, except that 2006 is not an exact multiple of 3 so we cannot convert it completely into the form 8^x. We will write it in following manner-

$2^{2006} = 8^{668} \times 4$.

Now, 8^{668} gives the remainder 1 when divided by 7 as we have seen in the previous problem. And 4 gives a remainder of 4 only when divided by 7. Hence the remainder when 2^{2006} is divided by 7 is the remainder when the product 1 × 4 is divided by 7.

Answer = 4

4. **What is the remainder when 25^{25} is divided by 9?**

Again $25^{25} = (18 + 7)^{25} = (18 + 7)(18 + 7)$…25 times = $18K + 7^{25}$

Hence remainder when 25^{25} is divided by 9 is the remainder when 7^{25} is divided by 9.

Now $7^{25} = 7^3 \times 7^3 \times 7^3$.. (8 times) × 7 = 343 × 343 × 343… (8 times) × 7.

The remainder when 343 is divided by 9 is 1 and the remainder when 7 is divided by 9 is 7.

Hence the remainder when 7^{25} is divided by 9 is the remainder we obtain when the product 1 × 1 × 1… (8 times) × 7 is divided by 9. The remainder is 7 in this case. Hence the remainder when 25^{25} is divided by 9 is 7.

Some Special Cases:

2.1A When both the dividend and the divisor have a factor in common.

Let N be a number and Q and R be the quotient and the remainder when N is divided by the divisor D.

Hence, N = Q × D + R.

Let N = k × A and D = k × B where k is the HCF of N and D and k > 1.

Hence kA = Q × kB + R.

Let Q_1 and R_1 be the quotient and the remainder when A is divided by B.

Hence $A = B \times Q_1 + R_1$.

Putting the value of A in the previous equation and comparing we get-

$k(B \times Q_1 + R_1) = Q \times kB + R \rightarrow R = kR_1$.

Hence to find the remainder when both the dividend and the divisor have a factor in common,

- Take out the common factor (i.e. divide the numbers by the common factor)
- Divide the resulting dividend (A) by resulting divisor (B) and find the remainder (R_1).
- The real remainder R is this remainder R1 multiplied by the common factor (k).

Examples

5. **What the remainder when 2^{96} is divided by 96?**

The common factor between 2^{96} and 96 is $32 = 2^{5}$.

Removing 32 from the dividend and the divisor we get the numbers 2^{91} and 3 respectively.

The remainder when 2^{91} is divided by 3 is 2.

Hence the real remainder will be 2 multiplied by common factor 32.

Answer = 64

2.1B THE CONCEPT OF NEGATIVE REMAINDER

$15 = 16 \times 0 + 15$ or

$15 = 16 \times 1 - 1$.

The remainder when 15 is divided by 16 is 15 the first case and -1 in the second case.

Hence, the remainder when 15 is divided by 16 is 15 or -1.

–> When a number N < D gives a remainder R (= N) when divided by D, it gives a negative remainder of R – D.

For example, when a number gives a remainder of -2 with 23, it means that the number gives a remainder of 23 – 2 = 21 with 23.

EXAMPLE

6. **Find the remainder when 7^{52} is divided by 2402.**

Answer: $7^{52} = (7^{4})^{13} = (2401)^{13} = (2402 - 1)^{13} = 2402K + (-1)^{13} = 2402K - 1$.

Hence, the remainder when 7^{52} is divided by 2402 is equal to -1 or 2402 – 1 = 2401.

Answer: 2401.

2 .1C When dividend is of the form $a^{n} + b^{n}$ or $a^{n} - b^{n}$:

Theorem1: $a^n + b^n$ is divisible by a + b when n is **ODD.**
Theorem 2: $a^n - b^n$ is divisible by a + b when n is **EVEN.**
Theorem 3: $a^n - b^n$ is ALWAYS divisible by a – b.

EXAMPLES

Topic: REMINDER

7. **What is the remainder when $3^{444} + 4^{333}$ is divided by 5?**

Answer:

The dividend is in the form $a^x + b^y$. We need to change it into the form $a^n + b^n$.

$3^{444} + 4^{333} = (3^4)^{111} + (4^3)^{111}$.

Now $(3^4)^{111} + (4^3)^{111}$ will be divisible by $3^4 + 4^3 = 81 + 64 = 145$.

Since the number is divisible by 145 it will certainly be divisible by 5.

Hence, the remainder is 0.

8. **What is the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7?**

Answer:

The remainders when 5555 and 2222 are divided by 7 are 4 and 3 respectively.

Hence, the problem reduces to finding the remainder when $(4)^{2222} + (3)^{5555}$ is divided by 7.

Now $(4)^{2222} + (3)^{5555} = (4^2)^{1111} + (3^5)^{1111} = (16)^{1111} + (243)^{1111}$.

Now $(16)^{1111} + (243)^{1111}$ is divisible by 16 + 243 or it is divisible by 259, which is a multiple of 7.

Hence the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7 is zero.

9. **$20^{2004} + 16^{2004} – 3^{2004} – 1$ is divisible by:**

(a) 317 (b) 323 (c) 253 (d) 91

Solution: $20^{2004} + 16^{2004} – 3^{2004} – 1 = (20^{2004} – 3^{2004}) + (16^{2004} – 1^{2004})$.

Now $20^{2004} – 3^{2004}$ is divisible by 17 (Theorem 3) and $16^{2004} – 1^{2004}$ is divisible by 17 (Theorem 2).

Hence the complete expression is divisible by 17.

$20^{2004} + 16^{2004} – 3^{2004} – 1 = (20^{2004} – 1^{2004}) + (16^{2004} – 3^{2004})$.

Now $20^{2004} – 1^{2004}$ is divisible by 19 (Theorem 3) and $16^{2004} – 3^{2004}$ is divisible by 19 (Theorem 2).

Hence the complete expression is also divisible by 19.

Hence the complete expression is divisible by 17 × 19 = 323.

2.1D When $f(x) = a + bx + cx^2 + dx^3 +…$ is divided by $x – a$

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The remainder when $f(x) = a + bx + cx^2 + dx^3 +..$ is divided by $x - a$ is $f(a)$.
So, If $f(a) = 0$, $(x - a)$ is a factor of $f(x)$.

EXAMPLES

10. **What is the remainder when $x^3 + 2x^2 + 5x + 3$ is divided by $x + 1$?**

Answer: The remainder when the expression is divided by (x – (-1)) will be f(-1).

Remainder = $(-1)^3 + 2(-1)^2 + 5(-1) + 3 = -1$

11. **If $2x^3 - 3x^2 + 4x + c$ is divisible by $x – 1$, find the value of c.**

Since the expression is divisible by x – 1, the remainder f(1) should be equal to zero.

Or 2 – 3 + 4 + c = 0, or c = -3.

2 .1E Fermat's Theorem

If p is a prime number and N is prime to p, then $N^p - N$ is divisible by p.

EXAMPLE

12. **What is the remainder when $n^7 – n$ is divided by 42?**

Answer: Since 7 is prime, $n^7 – n$ is divisible by 7.

$n^7 – n = n(n^6 – 1) = n (n + 1)(n – 1)(n^4 + n^2 + 1)$

Now (n – 1)(n)(n + 1) is divisible by 3! = 6

Hence $n^7 – n$ is divisible by 6 x 7 = 42.

Hence the remainder is 0.

2.1F Wilson's Theorem

If p is a prime number, (p – 1)! + 1 is divisible by p.

EXAMPLE

13. **Find the remainder when 16! Is divided by 17.**

16! = (16! + 1) -1 = (16! + 1) + 16 – 17

Every term except 16 is divisible by 17 in the above expression. Hence the remainder = the remainder obtained when 16 is divided by 17 = 16

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Answer = 16

2.1G TO FIND THE NUMBER OF NUMBERS, THAT ARE LESS THAN OR EQUAL TO A CERTAIN NATURAL NUMBER N, AND THAT ARE DIVISIBLE BY A CERTAIN INTEGER

To find the number of numbers, less than or equal to n, and that are divisible by a certain integer p, we divide n by p. The quotient of the division gives us the number of numbers divisible by p and less than or equal to n.

EXAMPLE

14. How many numbers less than 400 are divisible by 12?

Answer: Dividing 400 by 12, we get the quotient as 33. Hence the number of numbers that are below 400 and divisible by 12 is 33.

15. How many numbers between 1 and 400, both included, are not divisible either by 3 or 5?

Answer: We first find the numbers that are divisible by 3 or 5. Dividing 400 by 3 and 5, we get the quotients as 133 and 80 respectively. Among these numbers divisible by 3 and 5, there are also numbers which are divisible both by 3 and 5 i.e. divisible by 3 x 5 = 15. We have counted these numbers twice. Dividing 400 by 15, we get the quotient as 26.

Hence the number divisible by 3 or 5 = 133 + 80 – 26 = 187

Hence, the numbers not divisible by 3 or 5 are = 400 – 187 = 213.

16. How many numbers between 1 and 1200, both included, are not divisible by any of the numbers 2, 3 and 5?

Answer: as in the previous example, we first find the number of numbers divisible by 2, 3, or 5. from set theory we have

n(AUBUC) = n(A) + n(B) + n(C) – n(A intersectn B) – n(B intersectn C) – n(A intersectn C) + n(A intersectn B intersectn C)

n(2U3U5) = n(2) + n(3) + n(5) – n(6) – n(15) – n(10) + n(30)

–> n(2U3U5) = 600 + 400 + 240 – 200 – 80 – 120 + 40 = 880

Hence number of numbers not divisible by any of the numbers 2, 3, and 5

= 1200 – 880 = 320.