Topic: Logarithms

In this article we will discuss about **Logarithms.** This topic are very useful and acts as the base of simplification and Algebra. It is compulsory that question will comes from this topic.

LOGARITHMS:

Remainder Theorem : Let p(x) be any polynomial of degree geater than or equal to one and 'a' be any real number. If p(x) is divided by (x – a), then the remainder is equal to p(a).

Factor Theorem : Let p(x) be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that p(a) = 0, then (x – a) is a factor of p(x). Conversely, if (x – a) is a factor of p(x), then p(a) = 0.

Note : Let p(x) be any polynomial of degree greater than or equal to one. If leading coefficient of p(x) is 1 then p(x) is called monic. (Leading coefficient means coefficient of hi

Important Formulas:

1. **Logarithm:**

If a is a positive real number, other than 1 and $a^m = x$, then we write:
$m = \log_a x$ and we say that the value of log x to the base a is m.

Examples:

(i). 10^3 1000 $\Rightarrow$ $\log_{10} 1000 = 3$.

(ii). $3^4 = 81$ $\Rightarrow$ $\log_3 81 = 4$.

(iii). $2^{-3} = \frac{1}{8}$ $\Rightarrow$ $\log_2 \frac{1}{8} = -3$.

(iv). $(.1)^2 = .01$ $\Rightarrow$ $\log_{(.1)} .01 = 2$.

2. **Properties of Logarithms:**

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_x x = 1$
4. $\log_a 1 = 0$

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5. $\log_a (x^n) = n(\log_a x)$

6. $\log_a x = \dfrac{1}{\log_x a}$

7. $\log_a x = \dfrac{\log_b x}{\log_b a} = \dfrac{\log x}{\log a}$.

3. **Common Logarithms:**
   Logarithms to the base 10 are known as common logarithms.

4. The logarithm of a number contains two parts, namely 'characteristic' and 'mantissa'.

   Characteristic: The internal part of the logarithm of a number is called its **characteristic.**

   Case I: When the number is greater than 1.
   In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

   Case II: When the number is less than 1.
   In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.
   Instead of -1, -2 etc. we write 1 (one bar), 2 (two bar), etc.

   Examples:-

Number	Characteristic	Number	Characteristic
654.24	2	0.6453	1
26.649	1	0.06134	2
8.3547	0	0.00123	3

Mantissa:

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The decimal part of the logarithm of a number is known is its **mantissa.** For mantissa, we look through log table.

LOGARITHM :

Definition : Every positive real number N can be expressed in exponential form as

$N = a^x$ ....(1) ***e.g.*** $49 = 7^2$

where 'a' is also a positive real different than unity and is called the base and 'x' is called the exponent.

We can write the relation (1) in logarithmic form as

$\log_a N = x$ ....(2)

Hence the two relations

$$\left.\begin{array}{r} a^x = N \\ \text{and} \quad \log_a N = x \end{array}\right]$$

are identical where $N > 0, a > 0, a \neq 1$

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number. Logarithm of zero does not exist and logarithm of (–) ve reals are not defined in the system of real numbers.

i.e a is raised what power to get N

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PRINCIPAL PROPERTISE OF LOGARITHM :

If m, n are arbitrary positive real numbers where

$a > 0 \; ; \; a \neq 1$

(1) $\log_a m + \log_a n = \log_a mn \qquad (m > 0, n > 0)$

Proof: Let $x_1 = \log_a m \quad ; \quad m = a^{x}$

$x_2 = \log_a n \quad ; \quad n = a^{x_2}$

Now $mn = a^{x} \; ; \; a^{x_2}$

$mn = a^{x_1 + x_2}$

$x_1 + x_2 = \log_a mn$

$\log_a m + \log_a n = \log_a mn$

(2) $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\frac{m}{n} = a^{x_1 - x_2}$

$x_1 - x_2 = \log_a \frac{m}{n}$

$\log_a m - \log_a n = \log_a \frac{m}{n}$